Useful Formulae

- # permutations of n different objects $= |S_n| = n!$
- # ways of choosing k things from n with replacement $= n^k$
- # " " " " " " " without replacement

$$= \frac{n(n-1)\dots(n-k+1)}{k!} = \frac{n!}{k!\,(n-k)!}$$

This is often denoted ${}^nC_k$ or $\binom{n}{k}$. Notice that numerically $\binom{n}{k} = \binom{n}{n-k}$

We could also think of this as a permutation of the n objects with the k to be chosen occupying the first k slots and the $(n-k)$ rejected being in the last slots. Hence this is the # permutations of n objects comprising 2 types (the 'chosen' and the 'others') with k of type 1 and $r = n-k$ of type 2. This could be notated

$$\binom{n}{k\ \ r} := \frac{n!}{k!\ r!} = \binom{n}{k} = \binom{n}{r}$$

- Since determining asymptotic behaviour of factorials gets messy, a useful approximation is

$$n! \sim \sqrt{2\pi}\, e^{-n}\, n^{n+1/2} \quad \text{(Stirling)}$$

- # permutations of $n = n_1 + \dots + n_t$ objects comprising t different types is $\binom{n}{n_1 \dots n_t} = \frac{n!}{n_1! \dots n_t!}$

e.g. # permutations of $\{a,a,a,b,b,c\}$ is $\frac{6!}{3!\,2!\,1!} = 60$

- $(x+y)^n = \sum_0^n \binom{n}{r} x^r y^{n-r}$
- $(x_1 + \dots + x_t)^n = \sum \binom{n}{r_1 \dots r_t} x_1^{r_1} \dots x_t^{r_t}$ with the sum taken over all $r_i \geq 0$ with $r_1 + \dots + r_t = n$