Now suppose x and y stat. indep., then for $z = x+y$,

$$\mu_z = E[x+y] = \mu_x + \mu_y \quad \text{(irrespective of independence)}$$

and

$$\begin{aligned}\sigma_z^2 = E[(z-\mu_z)^2] &= E[(x-\mu_x+y-\mu_y)^2] \\ &= E[(x-\mu_x)^2 + (y-\mu_y)^2 + 2(x-\mu_x)(y-\mu_y)] \\ &= \sigma_x^2 + \sigma_y^2 + 2E[x-\mu_x].E[y-\mu_y] = \sigma_x^2+\sigma_y^2.\end{aligned}$$

Actually, the mean and variance are just two examples of a collection of characteristics of distributions.

Definitions

The $\underline{k^{th} \text{ moment}}$ μ_k' of a distribution is given by

$$\mu_k' := E[x^k],$$

so the mean is the 1^{st} moment. It's actually more helpful to consider horizontally normalised shape information, hence we define the $\underline{k^{th} \text{ central moment}}$ μ_k of a distribution by

$$\mu_k := E[(x-\mu)^k],$$

so $\mu_1 = 0$ and μ_2 is the variance. There are some colourfully named descriptors, such as

$$\underline{\text{skewness}} := \frac{\mu_3}{\mu_2^{3/2}} \quad \text{and} \quad \underline{\text{kurtosis}} := \frac{\mu_4}{\mu_2^2}.$$

Notice that the ordinary moments determine the central moments...

$$\mu_2 = E[(x-\mu)^2] = E[x^2] - (E[x])^2 = \mu_2' - \mu^2,$$

and

$$\begin{aligned}\mu_3 = E[(x-\mu)^3] &= E[x^3] - 3E[x^2]\mu + 3E[x]\mu^2 - \mu^3 \\ &= \mu_3' - 3\mu_2'\mu + 2\mu^3.\end{aligned}$$