Eg. Roll a pair of dice $n = 36$ times and record
$X = \#$ pairs of 1's.

$p(\text{pair of 1's in one roll}) = \frac{1}{36} = p$

So

$$p(X = x, \text{binomial}) = \binom{36}{x}\left(\frac{1}{36}\right)^{x}\left(1 - \frac{1}{36}\right)^{36-x}$$

$$p(X = x, \text{Poisson}) = \frac{1^x e^{-1}}{x!} \quad \text{with } \lambda = 36 \cdot \frac{1}{36} = 1$$

so that we can expect 1 event in 36 rolls.

To illustrate ...

x	binomial	Poisson
0	0.363	0.368
1	0.373	0.368
2	0.187	0.184
3	0.060	0.061
4	0.014	0.015
5	0.003	0.003
6	0.000	0.001

But note also that $S_{\text{binomial}} = \{0, 1, 2, \ldots, 36\}$
and $S_{\text{Poisson}} = \{0, 1, 2, \ldots\}$.

Eg. Phone calls arrive on average at 2 every 3 minutes. Assuming Poisson, find $p(2 \text{ calls in } 9 \text{ minutes})$ and $p(\geq 5 \text{ calls in } 9 \text{ minutes})$.

Let $X = \#$ calls in 9 minutes — our 'unit interval'.
Here we'd expect 6 calls, so $\lambda = 6$. So

$$p(X = 2) = \frac{6^2 e^{-6}}{2!} = 18 e^{-6}$$

and

$$p(X \geq 5) = 1 - p(X \leq 4)$$
$$= 1 - \sum_{0}^{4} \frac{6^x e^{-6}}{x!}$$
$$\simeq 0.715$$