E.g. Roll a die until see 2nd "6", then $r=2$, and

$$P\begin{pmatrix}\text{see exactly 10}\\ \text{non-sixes before}\\ \text{see second 6}\end{pmatrix} = \binom{10+2-1}{1}\left(\tfrac{1}{6}\right)^2\left(\tfrac{5}{6}\right)^{10}$$

$P(X=10)$

$$\cong 0.049$$

Now

$$F(\infty) = \sum_0^\infty f(x) = \sum_{x=0}^\infty \binom{r+x-1}{r-1} p^r q^x$$

Notice that if $g(q) = (1-q)^{-t}$ then

$$g'(q) = t(1-q)^{-t-1}, \ldots, g^{(n)}(q) = \frac{(t+n-1)!}{(t-1)!}(1-q)^{-t-n}, \ldots$$

so

$$(1-q)^{-t} = \sum_0^\infty \frac{g^{(n)}(0)}{n!} q^n = \sum_0^\infty \binom{t+n-1}{n} q^n$$

Also, $\binom{r+x-1}{r-1} = \binom{r+x-1}{x}$

So $$F(\infty) = p^r \sum_{x=0}^\infty \binom{r+x-1}{x} q^x$$

$$= p^r(1-q)^{-r} = p^r(1-(1-p))^{-r} = 1$$

Hence $F(x)$ is called the <u>negative binomial distribution</u>.

- If n is large and p is close to zero (or 1), then the binomial distribution can be approximated by the Poisson distribution. (If p isn't close to 0 or 1 then we use the normal distribution as approximator.)

The <u>Poisson distribution</u> is given by

$$S = \{0, 1, 2, \ldots\} \text{ and } f(x) = \frac{\lambda^x e^{-\lambda}}{x!} \text{ for } \lambda > 0.$$

Notice that

$$F(\infty) = \sum_0^\infty f(x) = e^{-\lambda}\sum_0^\infty \frac{\lambda^x}{x!} = e^{-\lambda}e^{\lambda} = 1.$$