- With the Bernoulli experiment used for the binomial distribution, this time let

$$X = \#\ \bar{A}\text{'s before the } 1^{st}\ A$$

i.e., the number of failures before the first success. Then

$$p(X = x) = (1-p)^x p = f(x).$$

Notice that

$$F(a) = \sum_{x \le a} f(x) = \sum_{x \le a} (1-p)^x p = p \sum_{x \le a} (1-p)^x,$$

hence this is called the <u>geometric distribution</u>.
Notice also that

$$F(\infty) = p \sum_0^\infty (1-p)^x = p \frac{1}{1-(1-p)} = 1,$$

so

$$p(X > a) = \sum_{a+1}^\infty (1-p)^x p = p(1-p)^{a+1} \sum_0^\infty (1-p)^x$$

$$= (1-p)^{a+1}$$

and

$$p(X \le a) = F(a) = 1 - (1-p)^{a+1}$$

<u>**E.g.**</u> Roll a regular die and let $A$ = "4 displayed".

$$p(1^{st}\ 4 \text{ seen on } 6^{th} \text{ trial}) = p(X=5)$$

$$= (5/6)^x (1/6) \cong 0.067$$

$$p(X \geq 5) = p(X > 4) = (1 - 1/6)^5 \cong 0.402$$

$$p(X \le 4) = p(X < 5) \cong 1 - 0.402 = 0.598$$

- This time, repeat the Bernoulli trials until have exactly $r$ $A$'s (successes), then if $X = \#\bar{A}$'s (failures) then $x + r$ = # trials to get precisely $r$ successes. So

$$p\begin{pmatrix}\text{exactly } r\ A\text{'s} \\ \text{takes } r+x \text{ tries}\end{pmatrix} = \underbrace{\binom{r+x-1}{r-1} p^{r-1} (1-p)^x}_{\text{get } r-1\ A\text{'s and } x\ \bar{A}\text{'s in } (r+x-1) \text{ trials}} \underbrace{p}_{\text{succeed in last trial}}$$

$$= \binom{r+x-1}{r-1} p^r q^x = f(x), \text{ note } r \text{ constant!}$$