Recall: a <u>probability measure</u> on $\mathcal{U}$ is $p : \mathcal{P}(\mathcal{U}) \to \mathbb{R}$ such that

(i) $p(A) \geq 0 \quad \forall A \subseteq \mathcal{U}$

(ii) $p(\mathcal{U}) = 1$

(iii) $p\left(\dot{\bigcup}_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} p(A_i)$, $A_i \cap A_j = \emptyset$ for $i \neq j$.

<u>Remarks</u>

$p(\bar{A}) = 1 - p(A)$, since $\mathcal{U} = A \dot{\cup} \bar{A}$

$p(\emptyset) = 0$, since $\mathcal{U} = \mathcal{U} \dot{\cup} \emptyset$

$A \subseteq B \Rightarrow p(A) \leq p(B)$, since $B = (B - A) \dot{\cup} A$

$p(A \cup B) = p(A) + p(B) - p(A \cap B)$

$$p(A \cup B \cup C) = p(A) + p(B) + p(C) - p(A \cap B) - p(A \cap C) - p(B \cap C) + p(A \cap B \cap C)$$

etc..

<u>Examples</u>

Since the probability of an outcome is the ratio of the measure of the (weighted) favourable set to the measure of the (weighted) universe of possibilities,

- Flip coin twice, then
  $$\mathcal{U} = \{(h,h), (h,t), (t,h), (t,t)\}$$
  and $p(\geq \text{head}) = 3/4$.
- Toss 3 dice, then $p(\text{all even}) = 3^3/216 = 27/216$
  and $p(\text{at least one odd}) = 1 - 27/216$
- Pick one card from a pack of 52, then
  $$p(\text{ace or spade}) = \frac{4}{52} + \frac{13}{52} - \frac{1}{52}$$