17 $\models (A \leftrightarrow B) \leftrightarrow (B \leftrightarrow A)$

18 $\models (A \to B) \wedge (C \to B) \leftrightarrow (A \vee C \to B)$

19 $\models (A \to B) \wedge (A \to C) \leftrightarrow (A \to B \wedge C)$

20 $\models (A \to B) \leftrightarrow (\neg B \to \neg A)$

21 $\models A \vee B \leftrightarrow B \vee A$

22 $\models A \wedge B \leftrightarrow B \wedge A$

23 $\models (A \vee B) \vee C \leftrightarrow A \vee (B \vee C)$

24 $\models (A \wedge B) \wedge C \leftrightarrow A \wedge (B \wedge C)$

25 $\models A \vee (B \wedge C) \leftrightarrow (A \vee B) \wedge (A \vee C)$

26 $\models A \wedge (B \vee C) \leftrightarrow (A \wedge B) \vee (A \wedge C)$

27 $\models A \vee A \leftrightarrow A$

28 $\models A \wedge A \leftrightarrow A$

29 $\models \neg(A \vee B) \leftrightarrow \neg A \wedge \neg B$

30 $\models \neg(A \wedge B) \leftrightarrow \neg A \vee \neg B$

31 $\models A \to B \leftrightarrow \neg A \vee B$

32 $\models A \to B \leftrightarrow \neg(A \wedge \neg B)$

33 $\models A \vee B \leftrightarrow \neg A \to B$

34 $\models A \vee B \leftrightarrow \neg(\neg A \wedge \neg B)$

35 $\models A \wedge B \leftrightarrow \neg(A \to \neg B)$

36 $\models A \wedge B \leftrightarrow \neg(\neg A \vee \neg B)$

37 $\models (A \leftrightarrow B) \leftrightarrow (A \to B) \wedge (B \to A)$

Actual derivations in propositional logic are given as sequences of logical expressions coupled with a rule justifying that expression:

rule p — the expression is a premise,

rule t — $\exists$ expressions $P_1, \dots, P_m$ preceding this expression Q in the sequence and $\models P_1 \wedge \dots \wedge P_m \to Q$.

Example

Show that $A \vee B,\ A \to C,\ B \to D \models C \vee D$.

the lines of the derivation on which this line depends

(1)	$P_1: A \to C$	rule p
(1)	$P_2: A \vee B \to C \vee B$	rule t and tautology 11 $\models P_1 \to P_2$
(3)	$P_3: B \to D$	rule p
(3)	$P_4: C \vee B \to C \vee D$	rule t and tautology 11 $\models P_3 \to P_4$
(1,3)	$P_5: A \vee B \to C \vee D$	rule t and tautology 7 $\models P_2 \wedge P_4 \to P_5$
(6)	$P_6: A \vee B$	rule p
(1,3,6)	$P_7: C \vee D$	rule t and tautology 1 $\models P_5 \wedge P_6 \to P_7$

Since we've used our (partial!) list of tautologies as stepping stones in our sequence of derivations, it's worth noting that some have aquired special names; e.g. tautology #1 is called <u>modus ponens</u>, and #2 <u>modus tollens</u>