Definition

We say that two wffs A and B are <u>equivalent</u> if they have the same truth values, i.e. if $\models A \Longleftrightarrow B$.

Examples

- Notice that $(P \Rightarrow Q)$ is equivalent to $(\neg P \vee Q)$.
- Also $P \vee (Q \wedge R)$ is equivalent to $(P \vee Q) \wedge (P \vee R)$.

Definition

The statement Q is a <u>consequence</u> of statements $P_1, \ldots, P_m$, written

$$P_1, \ldots, P_m \models Q$$

if $\forall$ truth assignments to the atomic statements within the P_i, Q has the value T whenever every P_i takes the value T. Notice that we don't care if Q is also T sometimes if not all the P_i are T!!

Notice also that
$$\begin{aligned}
& P_1, \ldots, P_m \models Q \quad \text{iff} \\
& P_1 \wedge \cdots \wedge P_m \models Q \quad \text{iff} \\
& P_1 \wedge \cdots \wedge P_{m-1} \models P_m \rightarrow Q \quad \text{iff} \\
& \models P_1 \wedge \cdots \wedge P_m \rightarrow Q .
\end{aligned}$$

Before we continue, we list a collection of tautologies which are easy to check and which are basic to the subject...

1 $\models A \wedge (A \rightarrow B) \rightarrow B$

2 $\models \neg B \wedge (A \rightarrow B) \rightarrow \neg A$

3 $\models \neg A \wedge (A \vee B) \rightarrow B$

4 $\models A \rightarrow (B \rightarrow A \wedge B)$

5 $\models A \wedge B \rightarrow A$

6 $\models A \rightarrow A \vee B$

7 $\models (A \rightarrow B) \wedge (B \rightarrow C) \rightarrow (A \rightarrow C)$

8 $\models (A \wedge B \rightarrow C) \rightarrow (A \rightarrow (B \rightarrow C))$

9 $\models (A \rightarrow (B \rightarrow C)) \rightarrow (A \wedge B \rightarrow C)$

10 $\models ((A \rightarrow B) \wedge \neg B) \rightarrow \neg A$

11 $\models (A \rightarrow B) \rightarrow (A \vee C \rightarrow B \vee C)$

12 $\models (A \rightarrow B) \rightarrow (A \wedge C \rightarrow B \wedge C)$

13 $\models (A \rightarrow B) \rightarrow ((B \rightarrow C) \rightarrow (A \rightarrow C$

14 $\models (A \leftrightarrow B) \wedge (B \leftrightarrow C) \rightarrow (A \leftrightarrow C$

15 $\models A \leftrightarrow A$

16 $\models A \leftrightarrow \neg\neg A$