

As an example, let's analyse the expression

$$(A \Rightarrow B) \Rightarrow (A \vee C \Rightarrow B \vee C)$$

using a truth table . . .

A	B	C	$A \Rightarrow B$	$A \vee C$	$B \vee C$	$A \vee C \Rightarrow B \vee C$	$(A \Rightarrow B) \Rightarrow (A \vee C \Rightarrow B \vee C)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	T	T	T
F	F	T	T	T	T	T	T
F	F	F	T	F	F	T	T

So our expression is permanently true, independent of the truth values of its component 'atomic' statements. Such an expression is called a tautology. There's a special notation for this, namely . . .

$$\models (A \Rightarrow B) \Rightarrow (A \vee C \Rightarrow B \vee C)$$

so " \models " means the expression following it is valid under all truth values.

Some expressions don't make sense, so

Definition

A well-formed formula (aka wff) is given by . . .

- (i) an atomic statement (either T or F value) is a wff,
- (ii) if A is a wff then $\neg A$ is a wff,
- (iii) if A and B are wffs then $A \vee B$, $A \wedge B$, $A \Rightarrow B$ and $A \Leftrightarrow B$ are wffs.