Examples

- $a0 = 0 \quad \forall a \in A$

  0 is unique, for if we assume $a + p = a \quad \forall a \in A$

  then $p = p + 0$ by axiom (iii)

  $= 0 + p$ by axiom (iv)

  $= 0$ by our assumption.

  $a + a0 = a1 + a0 = a(1+0) = a1 = a$

  so $a0$ = the unique 0.

- $a + a = a \quad \forall a \in A$

  $a + a = (a+a)1 = (a+a)(a+\bar{a})$

  $= a + a\bar{a}$ by axiom (x)

  $= a + 0 = a$

- $a + 1 = 1 \quad \forall a \in A$

  $a + 1 = a + (a + \bar{a}) = (a+a) + \bar{a} = a + \bar{a} = 1$

- $aa = a \quad \forall a \in A$

  $aa = aa + 0 = aa + a\bar{a} = a(a + \bar{a})$ by axiom (ix)

  $= a1 = a$

Many other similar results can be shown simply. There is also the very useful <u>principle of duality</u> which observes that any theorem of Boolean algebra has an equally valid dual theorem where addition and multiplication have been swapped, as have 0 and 1. This holds because our list of axioms contains its dual statements, so any proof of the original theorem can be converted into a proof of its dual result by exchanging each step in the proof by its dual.