Before we leave this topic, we should give two formal definitions which help to clarify our manipulations within predicate calculus. To substitute a variable y for a variable x in a formula P means to replace each free occurence of x in P by y. A formula $P(x)$ is free for y if no free occurence of x in $P(x)$ is in the scope of $(\forall y)$ or $(\exists y)$.

Examples

For $A(x) = P(x,y) \wedge (\forall y) Q(y)$, $A(x)$ is free for y.

For $B(x) = (x=1) \wedge (\exists y)(y \neq x)$, $A(x)$ is not free for y

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Definition

A Boolean algebra is a non-empty set A together with two binary operations (addition and multiplication) and a unary operation (complement, denoted $\bar{a}$) such that

(i) $a+b,\ ab,\ \bar{a} \in A \quad \forall a,b \in A$ (the operations are closed)

(ii) $a+(b+c) = (a+b)+c \quad \forall a,b,c \in A$

(iii) $\exists\ 0 \in A$ with $a+0=a \quad \forall a \in A$

(iv) $a+b = b+a \quad \forall a,b \in A$

(v) $a(bc) = (ab)c \quad \forall a,b,c \in A$

(vi) $\exists\ 1 \in A$ with $a1 = a \quad \forall a \in A$

(vii) $ab = ba \quad \forall a,b \in A$

(viii) $a+\bar{a} = 1$ and $a\bar{a} = 0 \quad \forall a \in A$

(ix) $a(b+c) = ab+ac \quad \forall a,b,c \in A$

(x) $a+(bc) = (a+b)(a+c) \quad \forall a,b,c \in A$

This relates to set theory by $\cup$ being $+$, $\cap$ being multiplication, set complement being $\bar{a}$, and $\emptyset$ being 0 with the universe being 1. For logic, the suite $\vee, \wedge, \neg, F, T$ correspond to $+, \times, \bar{a}, 0, 1$. Notice that these 'models' allow me to see the reasons behind (viii) and (x).