We can formalize the technique of 'proof by contradiction'. We define a set $\{P_1, \ldots, P_m\}$ of statements to be <u>satisfiable</u> iff there exists at least one assignment of truth values to the prime components of the P_i so that all the P_i are simultaneously T. A <u>contradiction</u> is a formula which always takes the value F.

Hence proof by contradiction amounts to ...

$$P_1, \ldots, P_m \models Q \quad \text{if} \quad P_1, \ldots, P_m, \neg Q \models \text{any contradiction}$$

provided that the set $\{P_1, \ldots, P_m\}$ is satisfiable. We can actually <u>prove</u> this! [Suppose $\{P_1, \ldots, P_m\}$ satisfiable and suppose $\exists$ some formula C for which $P_1, \ldots, P_m, \neg Q \models C \wedge \neg C$.

Assign truth values to the prime components of the P_i so that they are all simultaneously T, then $P_1, \ldots, P_m \models \neg Q \rightarrow (C \wedge \neg C)$ and so $\neg Q \rightarrow (C \wedge \neg C)$ is T.

But $(C \wedge \neg C)$ is F, hence $\neg Q$ must be F, and so Q is T.]

<u>Example</u>

Show that $\{A \leftrightarrow B, B \rightarrow C, \neg C \vee D, \neg A \rightarrow D, \neg D\}$ is not satisfiable.

(1)	$A \leftrightarrow B$	p	
(2)	$B \rightarrow C$	p	
(3)	$\neg C \vee D$	p	
(4)	$\neg A \rightarrow D$	p	
(5)	$\neg D$	p	
(6)	$\neg\neg A$	t	(4,5)
(7)	A	t	(6)
(8)	$A \rightarrow C$	t	(1,2)
(9)	C	t	(7,8)
(10)	$\neg C$	t	(3,5)
(11)	$C \wedge \neg C$	t	(9,10)