# Formal Logic

At the start of the course we introduced the basic elements of mathematical logic, exploring manipulations of expressions and the equivalence with set theory. It's now time to formalise all that with definitions and language.

## Definitions

A logical _statement_ is permitted only one of two values; either true $T$ or false $F$. (There are useful logics allowing other options, but we will not be studying those in this course.)

We can use tables of truth values to define the basic _connectives_ of _propositional logic_. In particular, we define $\neg$ (not), $\wedge$ (and), $\vee$ (or), $\Rightarrow$ (implies), $\Leftrightarrow$ (iff) as follows. Let $P$ and $Q$ be logical statements, then

| $P$ | $\neg P$ |
|---|---|
| T | F |
| F | T |

| $P$ | $Q$ | $P \wedge Q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | F |

| $P$ | $Q$ | $P \vee Q$ |
|---|---|---|
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

| $P$ | $Q$ | $P \Rightarrow Q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

| $P$ | $Q$ | $P \Leftrightarrow Q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | T |

We can use these tables to analyse more complicated expressions.