- How many different cubes can we make if we paint 2 faces white, 2 faces red and 2 faces blue?
  - Again let A be the set of all colourings, then $|A| = \binom{6}{2}\binom{4}{2} = 90$. The relevant group is the set of o.p. symmetries, and $|G| = 24$. $|Fix(1)| = 90$. (Look again at page 24 for the group G.)

    Let r be a rotation of order 2. If it's a 'face rotation' then one pair of faces will be fixed and the other 4 swapped in pairs. Hence the swapped faces must be of the same colour, so $|Fix(r)| = 3.2 = 6$. If however it's an 'edge rotation' then no faces are fixed, so all 6 are swapped in pairs. So again $|Fix(r)| = 3.2 = 6$.

    Let r' be a rotation of order 3, then the orbit of any face has size 3, so we'd need to be able to paint 3 faces the same colour for a design to be fixed by r'. Hence $|Fix(r')| = 0$.

    Let r'' be a rotation of order 4, then 2 faces are fixed but the other four are in an orbit of size 4, which would need to be the same colour. Hence $|Fix(r'')| = 0$.

    So $N = \frac{1}{24}(90 + 9.6 + 8.0 + 6.0) = 6$.

- A flag has 6 horizontal bands (of the same width) where adjacent bands are allowed to have the same colour. If we have 3 colours to choose from for each band, how many different flags are there?
  - Let $A = \{(c_1, \ldots, c_6)\}$ be the set of all colourings. Here two flags are indistinguishable if one can be obtained from the other by reversing the order of the colours, so the relevant group $G = \{1, f\}$ where f is the reflection (180° flag rotation).

    $|A| = 3^6$, so $|Fix(1)| = 3^6$.

    f fixes only palindromes, so $|Fix(f)| = |\text{palindromes}| = 3^3$.

    Hence $N = \frac{1}{2}(3^6 + 3^3) = 378$.