- Given a circular table and six people, how many different seating arrangements are there?
  - Two seating arrangements are the same if they differ by a rotation, so with 6 people this means the corresponding group G has order 6 (namely $1, r, \ldots, r^5$ where r is rotation $60°$). Let A be all the arrangements, then $|A| = 6!$, and $|Fix(1)| = 6!$, $|Fix(r^i)| = 0$ for $1 \leq i \leq 5$, so $N = \frac{1}{6}(6! + \underbrace{0 + \ldots + 0}_{5 \text{ times}}) = 120$.

- Given 3 colours, how many different ways are there of painting the vertices of a square in 3-dimensional space?
  - Since we're in 3-dimensions, we can move the square both by rotating around and by flipping over (equivalent to the act of reflection in 2-dimensions), so two designs will be the same if they differ by an element of D_4 (page 23). Since we have 3 colours available for each vertex, the set A of all designs has size $3^4 = 81$. Recall that $|D_4| = 8$.
    Clearly $|Fix(1)| = 81$.
    Let r be a rotation of order 4 and f a diagonal reflection, so the other rotations are r^2, r^3 and reflections r^2f, rf, r^3f.
    Then $|Fix(r)| = |Fix(r^3)| = 3$, i.e. those designs having each vertex the same colour.
    $|Fix(r^2)| = 9$, i.e. those designs where diagonally opposite vertices have the same colour.
    $|Fix(f)| = |Fix(r^2f)| = 3^3 = 27$, since f and r^2f each fix their corresponding diagonal vertices (they reflect through that line), the colours of the non-fixed vertices must be the same, but each of the two fixed vertices can have any colour.
    $|Fix(rf)| = |Fix(r^3f)| = 3^2 = 9$ to match mirrored vertices.
    Hence $N = \frac{1}{8}(81 + 3 + 3 + 9 + 27 + 27 + 9 + 9) = 21$.