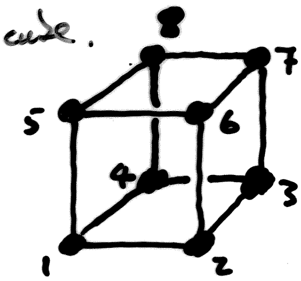
- More generally, we define the <u>dihedral group</u> D_n to be the group of isometries of a regular polygon with n sides. Then $|D_n| = 2n$, and the elements of D_n can be generated from one rotation and one reflection given that

$$r^n = 1, \quad f^2 = 1, \quad f r f^{-1} = r^{-1}.$$

Given that the full group of permutations S_n on n points has order $n!$ and D_n has order $2n$, we see that as n increases, D_n becomes an ever smaller subgroup of S_n.

- Let's play now in 3 dimensions. Since we're mostly concerned with concrete applications, noticing that reflection for 2-dimensional objects is equivalent to rotation in a 3rd dimension leads us to recognise that actually performing a reflection of a 3-dimensional object can be a bit tricky! So we define an <u>o.p. symmetry</u> to be one which does not require reflections. (Here, "o.p." stands for orientation preserving.) Consider the o.p. symmetries of a cube.



On page 22 we saw $|G| = |O(a)||G_a|$. This can be extended easily to subsets of A to give $|G| = |O(B)||G_B|$ for $B \subseteq A$. For example, if B is the face $\{2,3,7,6\}$ then G_B has 4 elements (rotations) and $O(B)$ has size 6 (the six faces), so $|G| = 6 \cdot 4 = 24$. Being more detailed...

- the identity (fixes all points)
- 3 rotations of order 2 (no fixed points) around the face centres
- 3 " " " 4 (no fixed points) " " " " ↺
- 3 " " " 4 (no fixed points) " " " " ↻
- 6 " " " 2 (no fixed points) " " edge centres
- 8 " " " 3 (2 fixed points) " " vertices.