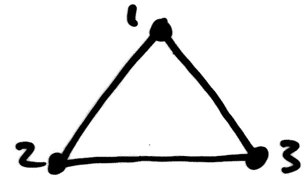
Examples

- To get some nice geometric pictures of group actions, we'll analyse groups of <u>symmetries</u> of an object, i.e. functions which move an object so that it occupies the same spot as it did originally. For example, the symmetries of an equilateral triangle comprise



  - the identity (fixes all points)
  - two rotations r_1, r_2 of order 3
    (these fix no points, r_1 rotates $120^\circ \circlearrowleft$ and $r_2 = r_1^2$)
  - three reflections f_1, f_2, f_3 of order 2
    (where f_i reflects through the line through the vertex i which bisects the opposite side, and hence fixes the one point i)

  If we let $A = \{1, 2, 3\}$ be the set of vertices, then
  - $\mathcal{O}(i) = A \quad \forall i \in A$, so G is transitive on A
  - $G_i = \{1, f_i\} \quad \forall i \in A$
  - $\mathrm{Fix}(1) = A$, $\mathrm{Fix}(r_i) = \phi$, $\mathrm{Fix}(f_i) = \{i\}$.

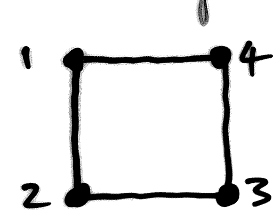
  If we denote the group of symmetries of this triangle by D_3, then we can generate all 6 elements of D_3 from just r_1 and f_1. In particular:

  $$1 = r_1^3 = f_1^2, \qquad f_1 = f_1,$$
  $$r_1 = r_1, \qquad f_2 = f_1 r_1,$$
  $$r_2 = r_1^2, \qquad f_3 = f_1 r_1^2.$$

  apply r_1 first then f_1

  Notice that D_3 is not Abelian, since $f_1 r_1 f_1^{-1} = r_1^{-1}$.

- Repeating the above for the group D_4 of symmetries of a square, we get



  - the identity (fixes all points)
  - 2 rotations of order 4 (no fixed points)
  - 1 " " " 2 " " "
  - 2 reflections " " 2 " " " through horiz/vertical.
  - 2 " " " 2 (2 fixed points) through diagonals