

We will consider this composition as a sequence of 'events' where each event depends on at most the result of the immediately preceding event. Such a process is called a Markov Chain and is an example of a stochastic process.

We denote the 'spectrum' of probabilities that in event e_i we are in the various states s_j by a vector of probabilities $\underline{v}_i = (p_1 \ p_2 \ \dots \ p_n)$, assuming here that there are n states. Of course, $p_1 + \dots + p_n = 1$. We list the transition probabilities in an $n \times n$ transition matrix...

$$M = \begin{pmatrix} q_{11} & q_{21} & \dots & q_{n1} \\ q_{12} & q_{22} & \dots & q_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ q_{1n} & q_{2n} & \dots & q_{nn} \end{pmatrix}$$

where $q_{ij} = \text{prob}(\text{next state} = s_j \mid \text{current state} = s_i)$.

It should be noted that authors are divided as to whether they write the matrix this way (where columns sum to 1) or the transpose of this (where rows sum to 1).

Notice that using matrix multiplication to compute the product $M \underline{v}_i$ gives a new vector whose j -th term is

$$q_{1j} p_1 + q_{2j} p_2 + \dots + q_{nj} p_n$$

which corresponds to adding $p(s_j | p_1) + p(s_j | p_2) + \dots + p(s_j | p_n)$

so gives the probability that we're now in state s_j . Hence $M \underline{v}_i = \underline{v}_{i+1}$

