19.A From Textbook 4.4-6

$$P(\text{ace or } \heartsuit) = P(\text{ace}) + P(\heartsuit) - P(\text{ace of } \heartsuit) = \frac{1}{13} + \frac{1}{4} - \frac{1}{13} \cdot \frac{1}{4} = \frac{16}{52} = \frac{4}{13} \approx 0.3077$$

19.B From Textbook 4.4-16

The first card of the hand can be any card.

$$\begin{aligned} P(\text{flush}) &= P(\text{second card is of the same suit}) \cdot P(\text{third card is OK}) \cdot P(\text{fourth}) \cdot P(\text{fifth}) \\ &= \frac{12}{51} \cdot \frac{11}{50} \cdot \frac{10}{49} \cdot \frac{9}{48} \\ &= \frac{11 \cdot 3}{17 \cdot 5 \cdot 49 \cdot 4} = \frac{33}{16660} \\ &\approx 1.98\ 10^{-3} \end{aligned}$$

Alternate method: select a suit (C_4^1) and all the possibles combinaison of 5 cards of that suit (C_{13}^5), the number of hands being C_{52}^5 and $P(\text{flush}) = \frac{C_4^1 \cdot C_{13}^5}{C_{52}^5} \approx 1.98\ 10^{-3}$

19.C From Textbook 4.4-28

Note: all the numbers are different, so once th first number has correctly been chosen, there are only 79 numbers left, among which only 10 have been selected by the lottery.

$$\begin{aligned} P(\text{winning}) &= P(\text{first number chosen is among the 11}) \\ &\quad \cdot P(\text{second number chosen is among the 11}) \\ &\quad \cdot P(\text{third number chosen is among the 11}) \\ &\quad \cdot \ldots \\ &\quad \cdot P(\text{seventh number chosen is among the 11}) \\ &= \frac{11}{80} \cdot \frac{10}{79} \cdot \frac{9}{78} \cdot \frac{8}{77} \cdot \frac{7}{76} \cdot \frac{6}{75} \cdot \frac{5}{74} \\ &= \frac{3}{28,879,240} \\ &\approx 1.0388\ 10^{-7} \end{aligned}$$

Alternate method: there are C_{11}^7 winning tickets out there, the total number of tickets being C_{80}^7 and $P(\text{flush}) = \frac{C_{11}^7}{C_{80}^7} \approx 1.0388\ 10^{-7}$

19.D From Textbook 4.4-32

The total number of possibles otcomes is $6 \cdot 6 = 36$ when there are two dice, $6 \cdot 6 \cdot 6 = 216$ when there are three dice.
The relevant pairs are {(2,6); (3,5); (4,4); (5,3); (6,2)}, that is 5.

The relevant triplet are {(1,1,6); (1,2,5); (2,1,5); (1,3,4); (2,2,4); (3,1,4); (1,4,3); (2,3,3); (3,2,3); (4,1,3); (1,5,2); (2,4,2); (3,3,2); (4,2,2); (5,1,2); (1,6,1); (2,5,1); (3,4,1); (4,3,1); (5,2,1); (6,1,1)}. There are 21 of them. BE careful to choose a way of ordering them. In my case the last number is decreasing, then the first one is increasing.
If you are smarter, only count those which are different modular a *circular* permutation: {(1,1,6); (1,2,5); (1,5,2); (1,3,4); (1,4,3); (2,2,4); (2,3,3) }. There are 7 of them, and each one counts for 3.

$$P(\text{making 8 with 2 dice}) = \frac{5}{36} \approx 0.13889$$
$$P(\text{making 8 with 3 dice}) = \frac{21}{216} \approx 0.09722$$

20.A From Textbook 4.5-6

$P(E) = 0.8$ and $P(F) = 0.6$.

$$\begin{aligned}
P(E \cap F) &= 1 - P(\overline{\overline{E \cap F}}) && \text{by complmentarity} \\
&= 1 - P(\overline{E} \cup \overline{F}) && \text{by De Morgan's law} \\
&= 1 - P(\overline{E}) - P(\overline{F}) + P(\overline{E} \cap \overline{F}) && \text{by Theorem 20.3} \\
&= 1 - (1 - P(E)) - (1 - P(F)) + P(\overline{E} \cap \overline{F}) && \text{by complementarity} \\
&= P(E) + P(F) - 1 + P(\overline{E} \cap \overline{F}) && \text{symplification} \\
&\geq P(E) + P(F) - 1 = 0.4 && \text{since } P(\overline{E} \cap \overline{F}) \geq 0
\end{aligned}$$

20.B From Textbook 4.5-10

We have $P(E) \cdot P(F) = P(E \cap F)$ and we want to show $P(\overline{E}) \cdot P(\overline{F}) = P(\overline{E} \cap \overline{F})$.

$$\begin{aligned}
P(\overline{E} \cap \overline{F}) &= 1 - P(\overline{\overline{E} \cap \overline{F}}) && \text{by complementary} \\
&= 1 - P(E \cup F) && \text{by De Morgan's law} \\
&= 1 - P(E) - P(F) + P(E \cap F) && \text{by Theoem 20.3} \\
&= 1 - P(E) - P(F) + P(E) \cdot P(F) && \text{by hypothesis} \\
&= (1 - P(E)) \cdot (1 - P(F)) && \text{factorisation} \\
&= P(\overline{E}) \cdot P(\overline{F}) && \text{by complementary}
\end{aligned}$$

20.C From Textbook 4.5-16

Let E by the event exactly four heads appear when a fair coin is flipped five times. Let F be the event the first flip was a tails. Then $P(F) = \frac{1}{2}$ and $P(E \cap F) = \frac{1}{2^5}$

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{1}{16}$$

20.D From Textbook 4.5-26ac

$$\begin{aligned} P(\text{no success}) &= (1-p)^n \\ P(\text{at most one success}) &= P(\text{no success}) + P(\text{exactly one success}) \\ &= (1-p)^n + np(1-p)^{n-1} \end{aligned}$$

21.A From Textbook 4.5-30

For those who are couragous, they can compute

$$E(N) = \sum_{n=0}^{n=10} n \cdot P(N=n) = \sum_{n=0}^{n=10} \frac{n \cdot C_{10}^{n} p^n (1-p)^{10-n}}{2^{10}}$$

For the others, it comes from $E(X+Y) = E(X) + E(Y)$ that $E(10X) = 10E(X)$. Let's call X the number of heads out of *one* flipping of our biaised coin. $E(X) = 0.6$ and

$$E(N) = 10E(X) = 6$$

21.B From Textbook 4.5-32

$$\begin{aligned} P(\text{winning}) = \frac{1}{C_{50}^{6}} &= \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2}{50 \cdot 49 \cdot 48 \cdot 47 \cdot 46 \cdot 45} \approx 6.29\ 10^{-8} \\ E &= \$\, P(\text{winning}) \cdot 10\ 10^6 + P(\text{loosing}) \cdot 0 - 1 \\ E &\approx \$\ -1 + 6.29\ 10^{-1} \\ E &\approx \$\ -0.37 \end{aligned}$$

21.C From Textbook 4.5-34

Same explanation as for 4.5-30 Recall that $E(\text{value of a die}) = \sum_{n=1}^{n=6} \frac{1}{6} n = \frac{21}{6} = \frac{441}{36} = 3.5$

$$E = 3E(\text{value of a die}) = 3 \cdot 3.5 = 10.5$$

21.D From Textbook 4.5-44

Same explanation as for 4.5-30, we have $V(10X) = 10V(X)$ where X is the random variable number of times a 6 is obtained when rolling of a fair die. df

$$V = 10V(X) = 10[E(X^2) - E(X)^2] = 10[E(X) - E(X)^2] = 10(\frac{1}{6} - \frac{1}{6}^2) = \frac{25}{18} \approx 1.3889$$