CS280 Homework 7	Solution Set

1.A Handout 17

A. 4.1-6

There are 4 routes from Boston to Detroit, 6 routes from Detroit to L.A. Each route from Boston to L.A. consists of choosing one route from a set of 4, and one route from a set of 6.

Thus there are $4 \cdot 6 = 24$ possible routes.

B. 4.1-12

The number of bit strings of length 6 or less is the sum of the numbers of bit strings of lengths $1, 2, 3, \ldots, 6$:

$$2 + 4 + 8 + 16 + 32 + 64 = \sum_{k=1}^{6} 2^k = 2(2^6 - 1) = 126.$$

If we count the empty string as a bit string, then the number is 127.

C. 4.1-16

To get the number of strings with an x in it, let's count all of those strings without any x, and subtract that from 26^4 (the number of all strings).

The number of strings with no x is 25^4: for each place in the string, we choose some letter which is not x. That's 25 possible letters for each place in the string.

Thus the answer is $26^4 - 25^4 = 66351$.

D. 4.1-18acd

(a)

The least positive integer less than 1000 which is divisible by 7 is 7. The greatest positive integer less than 1000 which is divisible by 7 is 994.

Every seventh number between 0 and 994 is also divisible by 7. Furthermore, these numbers constitute all of the positive integers less than 1000 which are divisible by 7.

Therefore, $\frac{994}{7} = 142$ is the answer.

(c)

Divisible by both 7 and 11

7 and 11 are relatively prime, so any number divisible by both 7 and 11 must have both in their prime factorization. We do the same basic trick as in part (a), but we divide by 11 as well, and choose a different upper bound.

The largest number divisible by both 7 and 11 which is less than 1000 is 924 $(7 \cdot 11 \cdot 12)$. We conclude that there are 12 numbers divisible by both 7 and 11 between 0 and 1000 (not counting 0).

(d)

Divisible by either 7 or 11

We determined the number divisible by 7 above, and it was 142. We add this number to the number divisible by 11, which is $990/11 = 90$ (since 990 is the largest integer divisible by 11 that is less than 1000).

The number of those divisible by both 7 and 11 is 12: 924 is the largest integer divisible by both 7 and 11 that is less than 1000, and $927/77 = 12$.

So the answer is 232 - 12 = 220. (We take the number divisible by 7 plus the number divisible by 11, and subtract the number divisible by both.)

E. 4.1-36

The size of the set is 100 elements. We calculate the number of sets with less than or equal to one element, then subtract this from 2^{100}.

The number of sets with no elements is 1, and the number of sets with one element is 100. Thus the answer is $2^{100} - 101$.

F. 4.1-46

The set of all uppercase and lowercase letters, digits, and the underscore has $2 \cdot 26 + 10 + 1$ elements.

Variable names determined by first eight characters, and the first character must be a letter. So the number of possible variable names of length 8 is $(2 \cdot 26) \cdot (2 \cdot 26 + 10 + 1)^7$.

We also have to account for shorter variable names. In general:
$\sum_{k=0}^{7}(2 \cdot 26) \cdot (2 \cdot 26 + 10 + 1)^k = (2 \cdot 26) \cdot \sum_{k=0}^{7}(2 \cdot 26 + 10 + 1)^k = 208130654417920$ is the number of possible variable names.

G. 4.2-2

Claim: If there are 30 students in a class, then at least 2 of them have last names that begin with the same letter.

Proof. *(Overly formal)* We define 26 “holes”, $H_1, \ldots, H_{26}$, where H_i contains all those students whose last name begins with the ith letter of the alphabet. Each student will be a “pigeon”. The number of pigeons is more than the number of holes, thus by the pigeonhole principle, there are two students in the same hole, i.e. their last names share the same first letter.

H. 4.2-16

There are 51 houses $H_1, \ldots, H_{51}$, each assigned a number between 1000 and 1099 (so there are 100 addresses). Let hole k be the hole containing only houses with either addresses $2k$ or $2k+1$ (so there are 50 holes). Let the set of houses $\{H_i\}$ be the pigeons. Thus we have 51 pigeons distributed over 50 holes. By the pigeonhole principle, there is a hole j where two pigeons reside. Thus there exist houses H_i and H_j such that one has address $2k$ and the other has $2k + 1$, for some k.

I. 4.2-34b

There are 9 students. Suppose it is not the case that there are at least three male students. Then, there are strictly less than three male students, call the number $k < 3$. The number of female students is $9 - k > 9 - 3 = 6$. Hence there must be more than 6 female students.

1.B Handout 18

A. 4.3-8

We have five runners, and no ties allowed. The number of orderings of this type is simply the number of ways to permute a list of five elements, i.e. $5! = 120$.

B. 4.3-14

In a set with 10 elements, for the subsets we choose either one element, three elements, five elements, seven elements, or nine elements. This amounts to $\sum_{k=0}^{4} C(10, 2k+1) = 512$ different subsets.

C. 4.3-32

The key insight here is that, by assigning a unique integer to each person sitting, we can treat a table arrangement which is "invariant under rotations of the table" as a list of integers, where adjacent integers in the list correspond to people sitting next to each other at the table. Furthermore, the person corresponding to the first integer on the list sits next to the person corresponding to the last integer on the list.

Note that for each list, every integer (1 through 6) appears exactly once, and for every list L of six integers, there are five others which are "equivalent" to L via a "rotation", (e.g. 123456, 234561, 345612, 456123, 561234, 621345 are all equivalent, in the sense that they represent the same table arrangement). With this interpretation in mind, the number of ways to seat six people at the table is $6!/6 = 120$.

D. 4.3-38

Applying the binomial theorem on p.256 directly, the coefficient of x^7 in $(1+x)^{11}$ is $C(11,7) \cdot 1^4 = \frac{11!}{7!(11-7)!} = \frac{11 \cdot 10 \cdot 9 \cdot 8}{4 \cdot 3 \cdot 2} = 11 \cdot 10 \cdot 3 = 330$.