8.A From Textbook 2.3 8ef

8ef 107 and 113 are primes. You only need to test the prime numbers up to $\lfloor\sqrt{n}\rfloor$. Since one can see that those numbers are not divisible by 2, 3, 5 nor 11, and that $70+35 = 15\times 7 = 105$, those numbers are prime (and you don't need a calculator to find it out).

8.B From Textbook 2.3 10ef

10ef $289 = 17^2$ and $899 = 29.31$ One can easily see that 2, 3, 5 and 11 do not divide those numbers. For 7: 280 is 7.4.10, and 910 is $700 + 210$ and 7 does not divide 9 nor -11. You just need to try 13 and above.

8.C From Textbook 2.3 28ab

28a $-17 \mod 2 = 1$ **28b** $144 \mod 7 = 140 + 4 \mod 7 = 4$

8.D From Textbook 2.3 46c

46c EAT DIM SUM

9.A From Textbook 2.4 2e

2e

$$\begin{aligned} & \gcd(1529, 14038) && = \gcd(277, 1529 \mod 277) && = \gcd(133, 11) \\ = & \gcd(1529, 14038 \mod 1529) && = \gcd(277, 144) && = \gcd(11, 1) \\ = & \gcd(1529, 277) && = \gcd(144, 133) && = 1 \end{aligned}$$

9.B From Textbook 2.4 8ac

8a $1\,1011_2 = 1\cdot 2^0 + 1\cdot 2^1 + 0\cdot 2^2 + 1\cdot 2^3 + 1\cdot 2^4 = 27$

8c

$$\begin{aligned} 11\,1011\,1110_2 &= 0\cdot 2^0 + 1\cdot 2^1 + 1\cdot 2^2 + 1\cdot 2^3 + 1\cdot 2^4 + 1\cdot 2^5 + 0\cdot 2^6 + 1\cdot 2^7 + 1\cdot 2^8 + 1\cdot 9^4 \\ \text{or (lazy way)} &= 100\,0000\,0000_2 - 1_2 - 100\,0000_2 - 1_2 \\ &= 2^{10} - 2^0 - 2^6 - 1 \\ &= 1024 - 1 - 64 - 1 = 958 \end{aligned}$$

9.C From Textbook 2.4 36

36

a :					1	1	1	0
b :					1	0	1	0
c_1 :					0	0	0	0
c_2 :				1	1	1	0	
c_3 :			0	0	0	0		
c_4 :		1	1	1	0			
$carry$:	1	1	1					
$a \cdot b$:	1	0	0	0	1	1	0	0

10.A From Textbook 2.5 2f

2f Note: $124 = 4.31$, $323 = 17.19$ and $\gcd(124, 323) = 1$

$323 = 2 \cdot 124 + 75$ $\quad 124 = 1 \cdot 75 + 49$

$75 = 1 \cdot 49 + 26$ $\quad 49 = 1 \cdot 26 + 23$

$26 = 1 \cdot 23 + 3$ $\quad 23 = 7 \cdot 3 + 2$

$3 = 1 \cdot 2 + 1$

$$\begin{aligned}
1 = 3 - 2 &= 3 - (23 - 7 \cdot 3) \\
&= (1+7) \cdot (26 - 1 \cdot 23) - 23 \\
&= (1+7) \cdot 26 + [(1+7) \cdot -1 - 1] \cdot 23 \\
&= (1+7) \cdot 26 - (1+7+1)(49 - 1 \cdot 26) \\
&= (1+7+(1+7+1)) \cdot (75 - 1 \cdot 49) - (1+7+1) \cdot 49 \\
&= 17 \cdot 75 - (17+9) \cdot (124 - 1 \cdot 75) \\
&= (17+26) \cdot (323 - 2 \cdot 124) - 26 \cdot 124 \\
&= 43 \cdot 323 - (2 \cdot 43 + 26) \cdot 124
\end{aligned}$$

$$\gcd(124, 323) = 43 \cdot 323 - 112 \cdot 124 = 1$$

10.B From Textbook 2.5 24ab

24a $\quad 3^4 \equiv 1 \pmod 5$, $3^6 \equiv 1 \pmod 7$, $3^{10} \equiv 1 \pmod{11}$

$$\begin{array}{lllll}
3^{302} \bmod 5 & = 3^{300} \cdot 3^2 \bmod 5 & = (3^4)^{75} \cdot 9 \bmod 5 & = 9 \bmod 5 & = 4 \\
3^{302} \bmod 7 & = 3^{300} \cdot 3^2 \bmod 7 & = (3^6)^{50} \cdot 9 \bmod 7 & = 9 \bmod 7 & = 2 \\
3^{302} \bmod 11 & = 3^{300} \cdot 3^2 \bmod 11 & = (3^{10})^{30} \cdot 9 \bmod 11 & = 9 \bmod 11 & = 9
\end{array}$$

24b One can see that 9 is solution: $9 \equiv 3^{302} \pmod 5$, $9 \equiv 3^{302} \pmod 7$ and $9 \equiv 3^{302} \pmod{11}$. $x \equiv 3^{302} \pmod{5 \cdot 7 \cdot 11}$ has a unique solution modulus 385, which has to verify the three previous relations. Those three have a unique solution modulus 385, (which is 9) due to the Chinese Remainder Theorem. Therefore, 9 is solution of $x \equiv 3^{302} \pmod{5 \cdot 7 \cdot 11}$.
If one is not convinced, there exists integers λ_1, λ_2, λ_3 such that $3^{302} - 9 = 5 \cdot \lambda_1 = 7 \cdot \lambda_2 = 11 \cdot \lambda_3 = \lambda$. Because 5,7 and 11 are relatively primes, $385 = 5 \cdot 7 \cdot 11 \mid \lambda$
and $3^{302} \bmod 385 = 9 + \lambda \bmod 385 = 9$.

If not, one can construct $x = a_1 M_1 y_1 + a_2 M_2 y_2 + a_3 M_3 y_3$ with $a_1 = 4$, $a_2 = 2$, $a_3 = 9$, $M_1 = 7 \cdot 11 = 77$, $M_2 = 5 \cdot 11 = 55$, $M_3 = 5 \cdot 7 = 35$, and y_k such that $M_k y_k \equiv 1 \pmod{m_k}$. Be careful not to consider another formula for y_k. The only simplification you can do is the following: $(M_k \bmod m_k) y_k \equiv 1 \pmod{m_k}$

For y_1: $77 \bmod 5 = 2$, $2 \cdot 3 = 6 \equiv 1 \pmod 5$ and $y_1 = 3$
For y_2: $55 \bmod 7 = 6$, $6 \cdot 6 = 36 \equiv 1 \pmod 7$ and $y_2 = 6$
For y_3: $35 \bmod 11 = 2$, $2 \cdot 6 = 12 \equiv 1 \pmod{11}$ and $y_3 = 6$

$x = 4 \cdot 77 \cdot 3 + 2 \cdot 55 \cdot 6 + 9 \cdot 35 \cdot 6 = 924 + 660 + 1890 = 3474 \equiv 9 \pmod{385}$

10.C From Textbook 2.5 26di

26d $a \bmod 4 = 2$, $a \bmod 7 = 1$
$a = 4k + 2 = 7l + 1 = 22$
(take $k = 5$ and $l = 3$)

26i $a \bmod 4 = 3$, $a \bmod 7 = 6$
$a = 4k - 1 = 7l - 1 = 27$
(take $k = 7$ and $l = 4$)