1.A From textbook Section 1.3: 6 and 20

$P(x,y)$ means student x has taken class y.

a) $\exists x \exists y P(x,y)$
There is a student in my class who has taken a class in computer science.

b) $\exists x \forall y P(x,y)$
There is a student in my class who has taken all computer science classes.

c) $\forall x \exists y P(x,y)$
Every student in my class has taken a class in computer science.

d) $\exists y \forall x P(x,y)$
There is a computer science class every body in my class has taken.

e) $\forall y \exists x P(x,y)$
For every class in the Computer Science Departement, there is a student from my class who has taken it. Or:
In each computer science class there is or has been a student from my class.

f) $\forall x \forall y P(x,y)$
Every student in my class has taken all computer science classes.

$Q(x,y) \;\Leftrightarrow\; x+y = x-y$

a)	$Q(1,1) =$ False	$2 = 1+1 \neq 1-1 = 0$
b)	$Q(2,0) =$ True	$2-0 = 2+0$
c)	$\forall y\ Q(1,y) =$ False	for example take $y=1$
d)	$\exists x\ Q(x,2) =$ False	this is equivalent to $4=0$
e)	$\exists x \exists y\ Q(x,y) =$ True	take $x=0$ and $y=1$
f)	$\forall x \exists y\ Q(x,y) =$ True	take $y=0$
g)	$\exists y \forall x\ Q(x,y) =$ True	take $y=0$
h)	$\forall y \exists x\ Q(x,y) =$ False	take $y=1$
i)	$\forall x \forall y\ Q(x,y) =$ False	take $x=2$ and $y=1$

1.B Move ¬ inside

$$\neg\forall x \exists y\ \big(A(x,y) \to B(x,y)\big)$$
$$\exists x \neg\exists y\ \big(A(x,y) \to B(x,y)\big) \qquad (\neg\forall x Z(x) \;\Leftrightarrow\; \exists x \neg Z(x))$$
$$\exists x \forall y\ \neg\big(A(x,y) \to B(x,y)\big) \qquad (\neg\exists y Z(y) \;\Leftrightarrow\; \forall y \neg Z(y))$$
$$\exists x \forall y\ \neg\big(\neg A(x,y)\ \vee\ B(x,y)\big) \qquad (A \to B \;\Leftrightarrow\; \neg A\ \vee B)$$
$$\exists x \forall y\ \big(A(x,y)\ \wedge\ \neg B(x,y)\big) \qquad (\neg(A \vee B) \;\Leftrightarrow\; \neg A\ \wedge \neg B)$$

1.C Show that $\forall x\big(A(x) \to B(x)\big) \to \big(\forall x A(x)\big) \to \big(\forall x B(x)\big)$ (*)

In other words:
assuming $\forall x\big(A(x) \to B(x)\big)$ (1), show $\big(\forall x A(x)\big) \to \big(\forall x B(x)\big)$ (2).

- When $\big(\forall x A(x)\big)$ is false, there is nothing to do, the proposition (2) holds.
- If $\big(\forall x A(x)\big)$ is true, then for every x A(x) is true, from (1) we have B(x) true, so $\big(\forall x B(x)\big)$ is true.

Therefore (*) holds.
The reverse doesn't hold, take A to be "$x > 2$" and B to be "$x > 4$", the universe of disclosure being $\mathbb{R}$

2.A From Textbook Section 1.4: 12

a) $|\emptyset| = 0$ **b)** $|\{\emptyset\}| = 1$ **c)** $|\{\emptyset, \{\emptyset\}\}| = 2$ **d)** $|\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}| = 3$
This is how Von Neumann constructed the set of integers $\mathbb{N}$ from scratch.

2.B From Textbook Section 1.5: 12de, 38

12 d) $(A - C) \cap (C - B) = \emptyset$

Let's take $x \in (A - C) \cap (C - B)$

- $x \in (A - C)$ so $x \in A$ and $x \notin C$
- $x \in (C - B)$ so $x \in C$ and $x \notin B$

So $x \in C$ and $x \notin C$! No such x exists.

12 e) $(B - A) \cup (C - A) = (B \cup C) - A$

$$\begin{aligned}
&(B - A) \cup (C - A) \\
&= \{x \mid x \in (B - A) \ \vee \ x \in (C - A)\} \\
&= \{x \mid \big(x \in B \wedge x \notin A\big) \ \vee \ \big(x \in C \wedge x \notin A)\} \\
&= \{x \mid \big(x \in B \vee x \in C\big) \ \wedge \ x \notin A\} \\
&= (B \cup C) - A
\end{aligned}$$

38) $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

a) $\{3, 4, 5\} \Rightarrow 0011100000$ **b)** $\{1, 3, 6, 10\} \Rightarrow 1010010001$
c) $\{2, 3, 4, 7, 8, 9\} \Rightarrow 0111001110$

3.A From Textbook Section 1.6: 16, 44, 48

16 $S = \{-1, 0, 2, 4, 7\}$

a) $f(x) = 1$
$f(S) = \{1\}$

b) $f(x) = 2x + 1$
$f(S) = \{-1, 1, 5, 9, 15\}$

c) $f(S) = \lceil x/5 \rceil$
$f(S) = \{0, 1, 2\}$

d) $f(S) = \lfloor (x^2 + 1)/3 \rfloor$
$f(S) = \{0, 1, 5, 16\}$

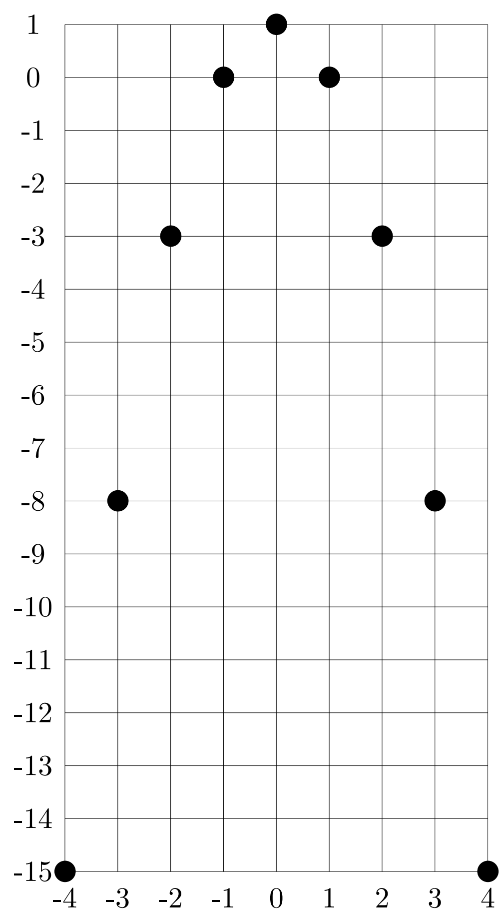
Don't forget to take the interger part, that $f(S)$ is a set, and don't write duplicate numbers in sets.

44 The number of bytes needed to encode n bits is $p = \lceil \frac{n}{8} \rceil$

a) $n = 4 \quad p = 1$ **b)** $n = 10 \quad p = 2$
c) $n = 500 \quad p = 63$ **d)** $n = 3000 \quad p = 375$

48 Graph of the function $f(n) = 1 - n^2$ from $\mathbb{Z}$ to $\mathbb{Z}$.
See graph on the right.

48 $f(n) = 1 - n^2$



3.B From Handout

$f(x) = x^3 - 1$, $g(x) = x + 1$, the set is $\mathbb{R}$.

$f \circ g(x) = f(g(x)) = f(x + 1) = (x + 1)^3 - 1$
$f \circ g(x) = x^3 + 3x + 3x$

Since $\sqrt[3]{(x^3 - 1) + 1} = \sqrt[3]{f(x) + 1} = x$
$f^{-1}(x) = \sqrt[3]{x + 1}$