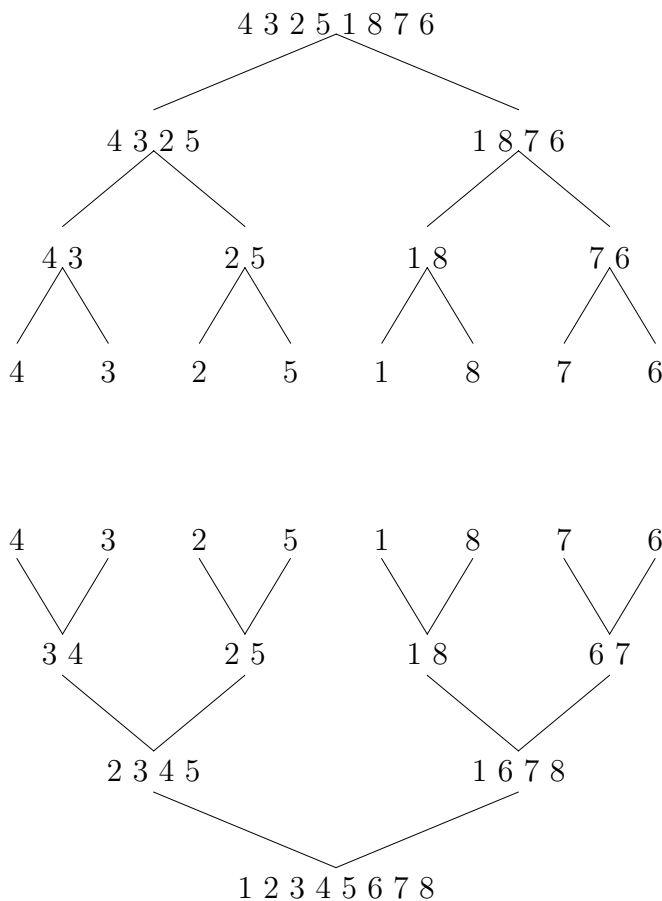


**36.A From Textbook 8.4-4**



Here are the different steps of the Merge sort: upper half, the splitting part; bottom half, merging while sorting the smaller lists.

**37.A From Textbook 9.1-6**

$$F(x, y, z) = xy + yz + zx$$

If at least two variables are true, say  $x$  and  $y$  (we do not care because of the symmetry of  $F$ :

$$F(x, y, z) = F(z, x, y) = F(y, z, x). \text{ then } F(1, 1, z) = 1 + z + z = 1$$

On the other hand, if at most one of the variable is true, then at least two of them are false, say  $x$  and  $y$ . Then  $F(0, 0, z) = 0 + z \cdot 0 + z \cdot 0 = 0$

**37.B From Textbook 9.1-20cd**

$$\text{Dual}(xyz + \bar{x}\bar{y}\bar{z}) = (x + y + z)(\bar{x} + \bar{y} + \bar{z})$$

$$\text{Dual}(x\bar{z} + x \cdot 0 + \bar{x} \cdot 1) = (x + \bar{z})(x + 1)(\bar{x} + 0)$$

**36.B From Textbook 4.4-6b**

I didn't understand the question as you did. Read the following if you want, or email Chang (cc144@cornell.edu) for the correct answer.

Merge sort run on the already sorted list 1 2 3 4 5 6 7 8 9 10 will do 14 comparisons, namely 1 and 2; 6 and 7 and the lowest level. Then at the second level of merging, 2 to merge 3 with 1 and 2, 2 to merge 8 with 6 and 7, and 1 to compare 4 with 5 and a last one to compare 9 with 10. At the next phase, 3 to merge 1 2 3 with 4 and 5, and 3 also to merge 6 7 8 with 9 and 10. And finally 5 to merge the two parts.

This is when the left half might have one more element than the right side when we split (as done in the book). In fact there are only 10 comparisons if when we split we put as many or one more on the right side.

**36.C From Textbook 8.4-8a**

I hope this is correct, email Chang (cc144@cornell.edu) for the correct answer.

When there is a single element on the left side, and 4 on the right side, we need 3 comparisons to merge in the worst case (doing a binary search).

**37.C From Textbook 9.2-4cd**

$x$	$y$	$z$	$F$	$x + y$
0	0	0	1	0
0	0	1	1	0
0	1	0	0	1
0	1	1	0	1
1	0	0	0	1
1	0	1	0	1
1	1	0	0	1
1	1	1	0	1

$x$	$y$	$z$	$F$	$xyz$
0	0	0	1	0
0	0	1	1	0
0	1	0	1	0
0	1	1	1	0
1	0	0	1	0
1	0	1	1	0
1	1	0	1	0
1	1	1	0	1

c)  $F(x, y, z) = 1$  if and only if  $x + y = 0$

sum of products:

$$\bar{x}\bar{y}z + \bar{x}\bar{y}\bar{z}$$

product of sums:

$$(x + \bar{y} + z)(x + \bar{y} + \bar{z})(\bar{x} + y + z) \cdot (x + \bar{y} + \bar{z})(\bar{x} + \bar{y} + z)(\bar{x} + \bar{y} + \bar{z})$$

d)  $F(x, y, z) = 1$  if and only if  $xyz = 0$

sum of products:

$$\bar{x}yz + x\bar{y}z + xy\bar{z} + \bar{x}\bar{y}z + x\bar{y}\bar{z} + \bar{x}y\bar{z} + \bar{x}\bar{y}\bar{z}$$

product of sums:

$$\bar{x} + \bar{y} + \bar{z}$$

**38.A From Textbook 9.3-4**

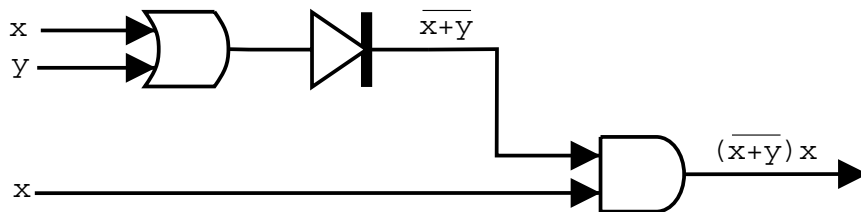
The output of the upper left AND gate is  $\bar{x}yz$ . Inverted, we have  $x + \bar{y} + \bar{z}$ . The output of the bottom left OR gate is  $\bar{x} + y + \bar{z}$ .

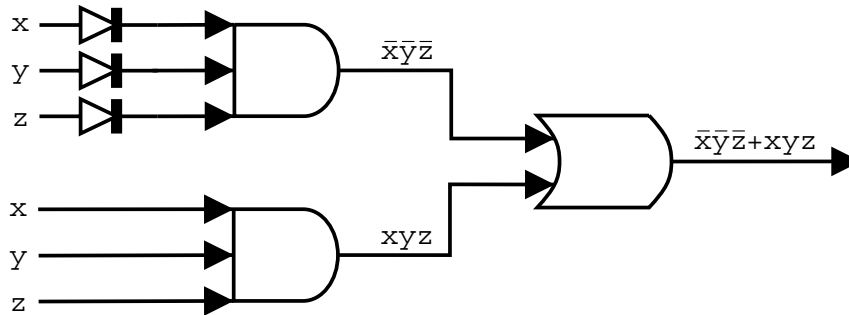
Combining both in an AND, we *gate*  $(x + \bar{y} + \bar{z})(\bar{x} + y + \bar{z}) = (x + \bar{y})(\bar{x} + y) + \bar{z}$

Note: The sign  $\bar{1}$  on your paper means that you didn't simplify  $\bar{x}\bar{y}\bar{z}$  in  $x + \bar{y} + \bar{z}$ , which you should do.

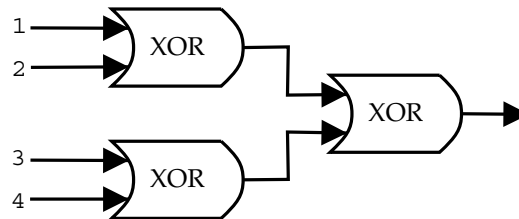
**38.B From Textbook 9.3-6bc**

The following circuit is correct, but if you think a little, there is one much simpler, which is to connect the output to a "0" or false, since the formula  $\overline{x + y}x = \bar{x}\bar{y}x = 0$ .





### 38.C From Textbook 9.3-8



How can one obtain an XOR gate? An XOR is an exclusive OR gate, that is 1 when  $x = 1$  and  $y = 0$  or when  $x = 0$  and  $y = 1$ , 0 otherwise. Basically, consider the output of a half adder: the sum  $s$ , and ignore the carry bit  $c$ .

Does this circuit does what we want? Yes. We want a 1 output when we switch any of the output. Consider the initial situation when all the inputs are 0, and the output 0. When switch one input, the output of the XOR gate it goes into changes value, and so does the last one.

At any time, if we change one input value, say 4. Whatever the value of 3 is, we change the output of their XOR gate. Therefore we flip the bottom input of the last XOR gate, and we flip the circuit output.

In more details, this gives: say 4 is  $x$ , and 3  $y$ . The output is  $o = x\bar{y} + \bar{x}y$ . If  $x$  was 1, and  $y$  0, then  $o = 1$ .  $x$  becomes 0, and  $o = 0$ . If  $y$  was 0,  $o$  was 0 and now is 1. The opposite if  $x$  was 0.

One other way to see the exercise is the design a circuit whose output is  $\bar{x}yzt + \bar{x}\bar{y}zt + xy\bar{z}t + xyz\bar{t} + \bar{x}\bar{y}\bar{z}t + \bar{x}y\bar{z}\bar{t} + \bar{x}y\bar{z}t + \bar{x}\bar{y}z\bar{t}$ . You can convince yourself that the output of the circuit is equivalent to the previous formula.