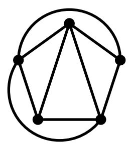
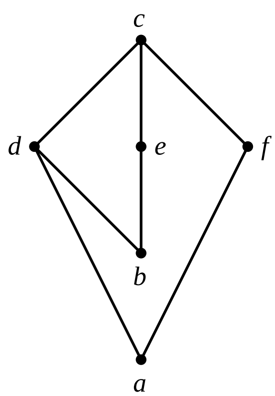
33A. Rosen 7.7-4



33B. Rosen 7.7-6

It is planar.



33C. Rosen 7.7-12

30 edges, 20 regions. Using Euler's formula, the number of vertices is $v = 30 - 20 + 2 = 12$.

33D. Rosen 7.7-18

The graph is planar: it can easily be made so by drawing the edge (a, h) around the offending edges and vertices it is currently crossing.

By Kuratowski's theorem, this graph does not contain a subgraph that is homeomorphic to $K_{3,3}$. Thus the graph itself cannot be homeomorphic to $K_{3,3}$.

33E. Rosen 7.8-4

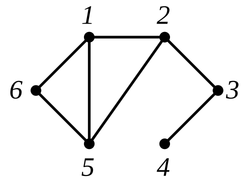
The chromatic number of this graph cannot be two. If it were, and g is colored "black," then b must be "white." But then a cannot be colored, as it has an edge to g and an edge to b. However, the graph can be colored with three colors. Color g green, color b, d, and f fuchsia[1], and color a, e, and c cerulean[2]. So the chromatic number is three.

33F. Rosen 7.8-16

As in Example 6, p. 516, draw a graph where vertices represent stations, and there is an edge between vertices if the two stations are within 150 miles of each other. Now we have an instance of graph coloring, where the minimum number of channels that can be placed is the chromatic number of the graph.

[1] reddish purple - named after the botanist Leonhard Fuchs

[2] sky blue



Looking at this graph, we immediately see that we need at least three channels, since vertices 1, 6, and 5 are all adjacent to each other. [The formal argument is essentially identical to the one used in 33E.] Now, we see that we can let 1 and 4 share channel one, 2 and 6 share channel two, and 3 and 5 share channel three. So three channels suffice.

[Interestingly, the distances in the table actually don't fit very well into two-dimensional euclidean space. For example, if we assume that station 1 is at $(0, 0)$ and that station 2 is at $(85, 0)$, then triangulation puts station 3 at $(130.7, 116.3)$ and station 4 at $(97.6, 174.5)$, which implies that the distance between 3 and 4 is really 67.0 instead of the expected 100. Similar problems crop up if you try three dimensions...]

34A. Rosen 8.1-2

a) vertex a is the root

b) vertices a, b, d, e, g, h, i and o are internal

c) vertices c, f, j, k, l, m, n, p, q, r, and s are leaves

d) vertices o and p are children of i

e) vertex d is the parent of h

f) vertex p is the sibling of o

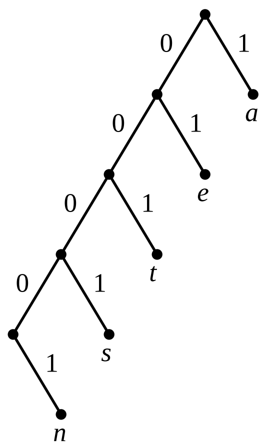
g) vertices g, b, and a are ancestors of m

h) vertices e, f, g, j, k, l, and m are descendants of b

34B. Rosen 8.1-10

If either m or n is 1, the graph will obviously be a tree, where all children have one common parent. A necessary condition for a graph to be a tree is that it must have one less edge than vertices. So, m and n must satisfy $mn = n + m - 1$. This reduces to $n(m - 1) = m - 1$. Thus, either $n = 1$ or $(m - 1) = 0$, i.e., $m = 1$. It can be seen easily that every complete bipartite graph with $m = 1$ or $n = 1$ will be a tree with all its children coming from one node.

34C. Rosen 8.2-12b



34D. Rosen 8.2-14d
The word is "tax."

```
01100101010
  t00101010
  t  a01010
  t  a    x
```

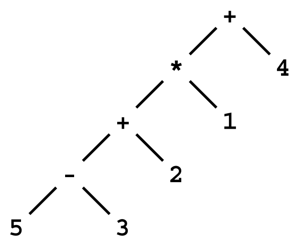
35A. Rosen 8.3-6b
No such tree exists. ← proof by contradiction: suppose that T is a [finite] ordered rooted tree whose leaves have the given universal addresses. Looking at the address list, we see that T has a suspicious-looking leaf at address 2.4.2.1; let s denote that leaf, let p denote s's parent (address 2.4.2), and let g denote s's grandparent (address 2.4). Now, p is g's second child [since p's address ends with a **2**], so p has a left sibling u at address 2.4.**1**. Since T is a finite tree, at least one of u's descendents (or perhaps u itself) must be a leaf, which means that at least one leaf address must begin with the sequence 2.4.1. This is a contradiction—no such address appears in the given list.

35B. Rosen 8.3-8 (preorder, inorder, postorder)

preorder:	a	b	d	e	i	j	m	n	o	c	f	g	h	k	l	p
inorder:	d	b	i	e	m	j	n	o	a	f	c	g	k	h	p	l
postorder:	d	i	m	n	o	j	e	b	f	g	k	p	l	h	c	a

35C. Rosen 8.3-26ac

a. $(((5-3)+2)*1)+4$



c. $(9/3)*((2*4)+(7-6))$