CS280 Homework 11	Solution Set

1.A Handout 28

28A. Rosen 6.5 - 2

(a) It's an equivalence relation.
(b) It's an equivalence relation.
(c) Is not transitive.
(d) Is not transitive.
(e) Is not transitive.

28B. Rosen 6.5 - 14(b)

Clearly, the matrix is symmetric (so the relation represented by it is symmetric).
There are all 1s on the diagonal, so the relation is reflexive.
Finally, the relation is not transitive. Let iRj correspond to the i, j entry of the matrix. The only non-reflexive relations between elements are $1R3$, $3R1$, $2R4$, $4R2$. Thus, if xRy and yRz, we conclude that either $x = y$, $y = z$ (one of the two represent reflexive relations, implying xRz trivially), or $x \neq y \neq z$ and $x = z$ (again implying xRz trivially).

28C. Rosen 6.5 - 22(b)(d)

We use the book's notation for this one.
(b) $[4]_3 = \{\dots, 4-2(3), 4-3, 4, 4+3, 4+2(3), \dots\} = \{\dots, -2, 1, 4, 7, 10, \dots\}$.
(d) $[4]_8 = \{\dots, 4-2(8), 4-8, 4, 4+8, 4+2(8), \dots\} = \{\dots, -12, -4, 4, 12, 20, \dots\}$.

28D. Rosen 6.6 - 2(b)

Reflexivity is clear.
Anti-symmetry: If xRy and yRx, then $x = y$, since the only relation that is not of the form iRi (corresponding to the i, i entry of the matrix) is $3R1$, and we do not have $1R3$.
Transitivity: is also obvious since if xRy and yRz, then either both are reflexive (implying xRz) or one of them is $3R1$ (both cannot be $3R1$). If it is xRy, then yRz must be $1R1$, and then xRz. If it is yRz, then xRy must be $3R3$, and then xRz.
So it's a partial order.

28E. Rosen 6.6 - 10(a)(c)

(a) All pairs less than $(2, 3)$: $\{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2)\}$.
(b) All pairs greater than $(3, 1)$: $\{(4,4), (4,3), (4,2), (4,1), (3,4), (3,3), (3,2)\}$.
(c) The 16-element Hasse diagram can be found on page 4.

28F. Rosen 6.6 - 24

(a) Maximal elements: l and m.
(b) Minimal elements: a, b, and c.
(c) No greatest element (two maximal elements).
(d) No least element (three minimal elements).
(e) Upper bounds of $\{a, b, c\}$: $\{k, l, m\}$.
(f) Least upper bound: k.
(g) Lower bounds of $\{f, g, h\}$: None.

(f) No greatest lower bound.

1.B Handout 29

29A. Rosen 7.1 - 4,6,8

(4) It's a **multigraph (and not a simple graph)**. Since it doesn't have any loops, we cannot claim it is not a multigraph.

(6) This is also a **multigraph (and not a simple graph)**.

(8) This is a **directed multigraph (and not a directed graph)**.

29B. Rosen 7.2 - 2

Number of vertices = 5, Number of edges = 13.

Degree of $a = 6$, degree of $b = 6$, degree of $c = 6$, degree of $d = 5$, degree of $e = 3$. (To verify this roughly, note that the sum of these is $26 = 2 \cdot 13$.)

There are no isolated vertices, nor are there pendant vertices.

29C. Rosen 7.2 - 18

The following are short sketches of how to prove these results. They are not meant to be rigorous. We did not require that you prove your answers.

(a) K_n is bipartite only if $n = 2$. This is because for $n = 1$, there is only one node in the graph (so by Rosen's definition, it cannot be bipartite); K_3 is not bipartite (see p.449), and the proof given there works for any K_n where $n \geq 3$.

(b) C_n is bipartite if and only if n is even. When n is even, we place every other node appearing on the cycle in a set S, and the rest of the nodes in a set T. It is easy to see that when n is even, there are no edges between nodes of S, nor are their edges between nodes of T. When n is odd, we cannot choose "every other node" as stated above.

(c) W_n is not bipartite for any $n \geq 3$. We have to place the "center node" of W_n in one of the two sets S or T. When we remove the center node from W_n, we have C_n, and any bipartition of C_n for $n \geq 3$ requires more than one node in both S and T (we take this as a fact implied by part b). But the center node has an edge to every node in the cycle C_n, so no matter which of the sets we choose, there will be a node in that set which has an edge to the center node.

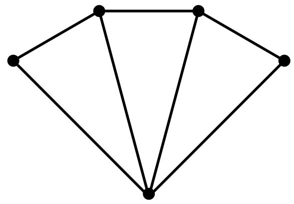
(d) Q_n is bipartite for all $n \geq 1$. It is clear that this is true for $n = 1$ and $n = 2$. We now outline the proof for $n \geq 3$. Define S as the set of nodes in Q_n with an even number of ones, and T as the set of nodes with an odd number of ones. For any $u, v \in S$, (u, v) is not an edge in the graph because if both u and v have an even number of ones but $u \neq v$, then the two nodes must differ in *at least two bits*. The same argument for any $u, v \in T$ shows that (u, v) is not an edge in the graph either.

29D. Rosen 7.2 - 20

The handshaking theorem says that $\sum_{v \in V} d(v) = 2|E|$, where V is the set of vertices, $d(v)$ is the degree of v, and E is the set of edges. So the number of edges is

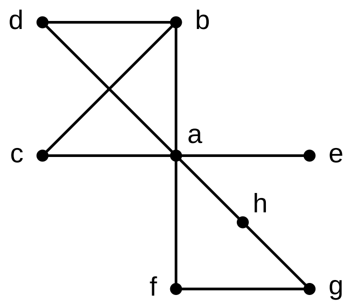
$$\frac{4+3+3+2+2}{2} = 14/2 = 7.$$

A graph with this property is given below.



29E. Rosen 7.2 - 32

One way to draw the union of these two graphs is the following:



Hasse diagram for 28E