CS280 HW10 Solutions

5.5 #8

Using inclusion-exclusion principle and the data given:
64 + 94 + 58 – 26 – 28 – 22 + 14 = 154.
So 154 students like at least one of the vegetables. There are 270 students in all, so 270 – 154 = 116 students do not like any of these vegetables.

5.5 #10

Note: "not exceeding " 100 means less than or equal to 100. So, there are 100 possible numbers, 1 to 100. floor(100/5) = 20 of them are divisible by 5. floor(100/7) = 14 of them are divisible by 7. floor(100/(5*7)) = 2 of them are divisible by both 5 and 7. By inclusion-exclusion, there are 20 + 14 – 2 = 32 numbers divisible by 5 or 7. Thus there are 100 – 32 = 68 numbers not divisible by 5 or 7.

5.5 #24

i. There are C(5,3) = 10 ways 3 tails can occur out of 5 coin flips.
ii. The possible outcomes are (T for tails, H for heads, x for either) TxxxT. Thus there are 2^3 = 8 ways for this to happen.
iii. The possible outcomes are xHxHx. There are again 2^3 = 8 ways for this to happen.

There are three sequences in which there are 3 tails and the first and last flips yield tails, TTHHT, THTHT, THHTT. There are 2 sequences in which the first and last flips are tails and the second and fourth are heads, THHHT and THTHT. There is one sequence where there are exactly 3 tails and the second and fourth flips are heads, THTHT. There is one sequence with exactly 3 tails, a tail in first and last position, and heads in the second and fourth position, THTHT. By inclusion-exclusion, there are 10 + 8 + 8 – 3 – 2 – 1 + 1 = 21 ways for any of i., ii., iii. to happen. There are 2^5 = 32 possible sequences of coin flips. So the probability is 21/32.

6.1 #4

a. This relation is antisymmetric and transitive. It is not reflexive: a cannot be taller than itself. It is not symmetric: if a is taller than b then b can't be taller than a. It is antisymmetric: if a is taller than b and b is taller than a then a must equal b (the fact that there are no such occurrences of a taller than b and b taller than a doesn't stop the relation from being antisymmetric). It is transitive: if a is taller than b and b is taller than c, clearly a is taller than c.
b. This relation is reflexive, symmetric, and transitive. a is born on the same day as a. If a is born on the same day as b, it must be that b is born on the same day as a. If a is born on the same day as b and b the same day as c, then clearly a is born on the same day as c. It is not antisymmetric: if a is born on the same day as b and b the same day as a, a and b need not necessarily be the same person.
c. This relation is reflexive, symmetric, and transitive. a has the same first name as itself. If a has the same first name as b, then b clearly has the same first name as a. If a has the same first name as b and b has the same first name as c, then clearly a has the same first name as c.

d. This relation is reflexive and symmetric. a has a common grandparent with itself. If a has a common grandparent with b, then clearly b has a common grandparent with a. It is not transitive: suppose a has a common grandparent with b, and b has a common grandparent with c. a's common grandparent with b might be b's grandparent on b's mother's side, while b's common grandparent with c might be on b's father's side; thus a and c need not necessarily have a common grandparent (imagine the family tree for this possibility). It is not antisymmetric: a and b need not be the same person if a has a common grandparent with b and b a common grandparent with a.

6.1 #8

Relation a is irreflexive, because a person cannot be taller than themselves. The other relations, all being reflexive, cannot be irreflexive.

6.1 #34

a. b. (all the same)

1	1	1	1	1		1	1	1	1	1
1	1	0	1	1		1	1	1	1	1
1	1	1	1	1		1	1	1	1	1
1	1	1	1	0		1	1	1	1	1
1	1	1	1	1		1	1	1	1	1

6.2 #4cd

c. The only field with every item being different from every other is Course number, so that is the primary key.

d. The only field with every item being different from every other is Departure time, so that is the primary key.

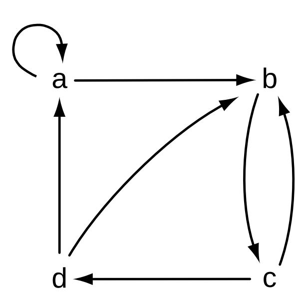
6.2 #10

The first table has 5 components. The second table has 8 components. The J3 operator combines these two tables via the last 3 components of the first table, and the first 3 components of the second table. Thus the resulting table will have 5 + 8 – 3 (inclusion-exclusion), or 10 components.

6.3 #8

a.	b.	c.	d.	e.
0 1 0	0 1 0	1 1 1	1 1 1	0 0 0
1 1 1	0 1 1	1 1 1	1 1 1	1 0 0
1 1 1	1 0 0	0 1 0	0 1 0	0 1 1
union	inter.	Compo.	Compo.	Sym. Diff.

6.3 #12



6.4 #20

a. When there is a city c such that there is a direct non-stop flight from a to c, and a direct non-stop flight from c to b.
b. When there are cities c, d such that there is a direct non-stop flight from a to c, a direct non-stop flight from c to d, and a direct non-stop flight from d to b.
c. When there are cities n1, n2, ….., n*, such that there is a direct non-stop flight from a to n1, . . ., and a direct non-stop flight from n* to b. (When there is a path in the transitive closure of R from a to b.)

6.4 #26bc

b.

A at start
```
0 0 0 0 0
0 0 1 0 1
0 0 0 0 1
1 0 0 0 0
0 1 1 0 0
```

A after i = 2 loop
```
0 0 0 0 0
0 1 1 0 1
0 1 1 0 0
0 0 0 0 0
0 0 1 0 1
```

A after i = 3 loop
```
0 0 0 0 0
0 1 1 0 1
0 0 1 0 1
0 0 0 0 0
0 1 1 0 1
```

A after i = 4 loop (and i = 5)
```
0 0 0 0 0
0 1 1 0 1
0 1 1 0 1
0 0 0 0 0
0 1 1 0 1
```

B (transitive closure)
```
0 0 0 0 0
0 1 1 0 1
0 1 1 0 1
1 0 0 0 0
0 1 1 0 1
```

c.
```
first step   second step   third step – can stop here, entries are 1s.
1 1 1 0 1    1 1 1 1 1     1 1 1 1 1
1 0 1 1 1    1 1 1 1 1     1 1 1 1 1
1 0 1 1 0    1 0 1 1 1     1 1 1 1 1
1 0 1 1 1    1 1 1 1 1     1 1 1 1 1
1 1 1 1 1    1 1 1 1 1     1 1 1 1 1
```