CS280 Homework 1
Solution Set by Ryan Williams

Preliminary note: I am trying to be very precise, since this is the beginning of the class and these logical aspects being covered are very crucial for understanding the material in the rest of the course. If you have any questions about these solutions, please email me at `rrw9@cornell.edu`.

All problems were graded out of 5 points.

A. 1.1

25c
Truth table for $(p \to q) \vee (\neg p \to r)$.

p	q	r	$(p \to q) \vee (\neg p \to r)$
T	T	T	$T\ T\ T\ \ T\ \ F\ T\ T$
T	T	F	$T\ T\ T\ \ T\ \ F\ T\ F$
T	F	T	$T\ F\ F\ \ T\ \ F\ T\ T$
T	F	F	$T\ F\ F\ \ T\ \ F\ T\ F$
F	T	T	$F\ T\ T\ \ T\ \ T\ T\ T$
F	T	F	$F\ T\ T\ \ T\ \ T\ F\ F$
F	F	T	$F\ T\ F\ \ T\ \ T\ T\ T$
F	F	F	$F\ T\ F\ \ T\ \ T\ F\ F$

25d
Truth table for $(p \to q) \wedge (\neg p \to r)$.

p	q	r	$(p \to q) \vee (\neg p \to r)$
T	T	T	$T\ T\ T\ \ T\ \ F\ T\ T$
T	T	F	$T\ T\ T\ \ T\ \ F\ T\ F$
T	F	T	$T\ F\ F\ \ F\ \ F\ T\ T$
T	F	F	$T\ F\ F\ \ F\ \ F\ T\ F$
F	T	T	$F\ T\ T\ \ T\ \ T\ T\ T$
F	T	F	$F\ T\ T\ \ F\ \ T\ F\ F$
F	F	T	$F\ T\ F\ \ T\ \ T\ T\ T$
F	F	F	$F\ T\ F\ \ F\ \ T\ F\ F$

42b
Exactly one is lying. Since the statements of Carlos and Diana contradict each other, one of them is the liar. Therefore, Alice is telling the truth, since there is only one liar. She says Carlos did it, so he did.

B. 1.2

12
We show $(\neg p \wedge (p \to q)) \to \neg q)$ is not a tautology. We use a truth table here. To see an example of a proof using identities, see the solution for 14.

p	q	$(\neg p \wedge (p \to q)) \to \neg q)$
T	T	$F\ F\ (T\ T\ T)\ T\ F$
T	F	$F\ F\ (T\ F\ F)\ T\ T$
F	T	$T\ T\ (F\ T\ T)\ F\ F$ $\leftarrow$ the proposition is false in this case, so not a tautology
F	F	$T\ F\ (F\ T\ F)\ T\ T$

The parentheses have been added simply for readability.

14

Show $p \leftrightarrow q \iff (p \wedge q) \vee (\neg p \wedge \neg q)$.

As in the previous problem, we can use identities or truth tables. We use identities because it is more informative to you, although it is longer than a truth table proof.

$p \leftrightarrow q \iff (p \rightarrow q) \wedge (q \rightarrow p)$ (definition)

$\iff (\neg p \vee q) \wedge (\neg q \vee p)$ (since $\neg A \vee B \iff A \rightarrow B$)

$\iff (\neg p \wedge \neg q) \vee (\neg p \wedge p) \vee (q \wedge \neg q) \vee (q \wedge p)$ (distributivity)

$\iff (\neg p \wedge \neg q) \vee F \vee F \vee (q \wedge p)$ (since $(\neg A \wedge A) \iff F$)

$\iff (\neg p \wedge \neg q) \vee (q \wedge p)$ (F is an identity for $\vee$)

$\iff (\neg p \wedge \neg q) \vee (p \wedge q)$ (commutativity)

$\iff (p \wedge q) \vee (\neg p \wedge \neg q)$ (commutativity)

(Note: The next two problems have informal proofs, since induction hasn't been introduced yet!)

28

Given a formula F using any of $\vee, \rightarrow, \leftrightarrow, \oplus$, we wish to rewrite F so that only $\neg$ and $\wedge$ are used. We do this as follows.

Wherever an expression of the form $P \vee Q$ occurs in F, we replace this expression with $\neg(\neg P \wedge \neg Q)$, which is an equivalent expression due to deMorgan's law.

If something of the form $P \rightarrow Q$ occurs in F, we replace the expression with $\neg(P \wedge \neg Q)$. This is equivalent due to the identity $P \rightarrow Q \iff \neg P \vee Q$ and deMorgan's law.

For $P \leftrightarrow Q$, we replace this expression with $\neg(P \wedge \neg Q) \wedge \neg(\neg P \wedge Q)$, since it is equivalent to $(P \rightarrow Q) \wedge (Q \rightarrow P)$.

For $P \oplus Q$, we use $\neg(\neg P \wedge \neg Q) \wedge \neg(P \wedge Q)$, since the expression is true when neither $P = Q =$ false, nor $P = Q =$ true.

The resulting formula after all of these replacements uses only $\neg$ and $\wedge$.

29

We know that using only $\neg$ and $\wedge$, we can express the other operators. Therefore, if we show that the $\wedge$ operator can be expressed using $\neg$ and $\vee$, then everything can be expressed using $\neg$ and $\vee$. But by deMorgan's law, $P \wedge Q \iff \neg(\neg P \vee \neg Q)$. So the algorithm is the following:

Given a formula F to convert to use only $\neg$ and $\vee$ operators:

(1) Use the proof of *28* to convert F to use only $\neg$ and $\wedge$ operators.

(2) Use deMorgan's law to change the $\wedge$s to expressions using $\neg$ and $\vee$.

The result is a formula using only $\neg$ and $\vee$.

C.

A straightforward use of Theorem 1.6 on the first handout yields:

$Q(p, q, r) = (\neg p \wedge \neg q \wedge r) \vee (p \wedge q \wedge \neg r)$.