1. Reading: K. Rosen *Discrete Mathematics and Its Applications*, 8.3
2. The main message of this lecture:

Tree traversal is a procedure for visiting all the vertices of a tree. There are some common traversal algorithms.

Definition 35.1. Universal address system for an ordered rooted tree labels all its vertices. Labels are n-tuples of nonnegative integers (separated by "."). They are defined recursively as follows:

1. The root r is labelled with 0 and its children are labelled $1, 2, \ldots, n$ left to right.
2. If an internal vertex $v \neq r$ has a label A, then its k_v children get labels $A.1,\ A.2,\ \ldots,\ A.k_v$ respectively.

Vertices in each finite tree can be then totally ordered by their universal addresses using the lexicographic ordering of their labels.

Examples 35.2. Slides!

Definition 35.3. Preorder traversal algorithm of an ordered rooted tree T is defined recursively.

1. If T consists of a root r only, then r is the preorder traversal of T.
2. Suppose $T_1, T_2, \ldots, T_n$ are the subtrees at r from left to right. The preorder traversal begins by visiting r, then T_1 in preorder, then T_2 in preorder, and so on, until T_n is traversed in preorder.

Examples 35.4. Slides!

Definition 35.5. Inorder traversal algorithm of an ordered rooted tree T is also defined recursively.

1. If T consists of a root r only, then r is the inorder traversal of T.
2. Suppose $T_1, T_2, \ldots, T_n$ are the subtrees at r from left to right. The inorder traversal begins by traversing T_1 in inorder, then visiting r. It continues by traversing T_2 in inorder, then T_3 in inorder, and so on, and finally T_n in inorder.

Examples 35.6. Slides!

Definition 35.7. Postorder traversal algorithm of an ordered rooted tree T is defined recursively as well.

1. If T consists of a root r only, then r is the postorder traversal of T.
2. Suppose $T_1, T_2, \ldots, T_n$ are the subtrees at r from left to right. The postorder traversal begins by traversing T_1 in postorder, then by traversing T_2 in postorder, then T_3 in postorder, $\ldots$, then T_n in postorder, and ends by visiting r.

Examples 35.8. Slides!

Definition 35.9. The standard fully parenthesized expression corresponds to the inorder traversal of the formation tree and is called the **infix form**. The **prefix form** corresponds to the preorder traversal of a formation tree. Sometimes it is also called the **Polish notation**. It is not too convenient for humans but is quite OK for computers since it provides a concise unambiguous notation. The **postfix form** of a term corresponds to the postorder traversal of the formation tree. Such expression is also said to be in the **reverse Polish notation**.

Examples 35.10. Slides!

Homework assignments. (The third installment due Friday 04/27)

35A:Rosen8.3-6b;   35B:Rosen8.3-8(preorder, inorder, postorder);   35C:Rosen8.3-26ac.