

Full name:

Student ID:

Statement of integrity: I did not, and will not, break the rules of academic integrity on this exam.

(Signature)

Show all your work. Calculators are allowed.

1. (10 points) A sequence X_n is defined recursively by

$$X_0 = a$$

$$X_1 = b, \text{ where } a, b \text{ are reals}$$

$$X_n = X_{n-1} + X_{n-2}, \text{ for } n > 1.$$

Prove by induction, that $X_n = b \cdot f_n + a \cdot f_{n-1}$ for all $n > 0$, where f_n is the n th Fibonacci number ($f_0 = 0$, $f_1 = 1$, $f_n = f_{n-2} + f_{n-1}$ for $n > 1$).

2. (10 points) How many different arrangements can be formed using the letters *Manhattan*?

3. (10 points) How many positive integers with five digits or less have neither their first digit equal to 3 nor their last digit equal to 5?

4. (10 points) Using Pascal's Triangle expand $(2x - y)^7$. Draw the corresponding portion of the Triangle. Feel free not to simplify the coefficients.

5. (10 points) How many positive integers are there less than 10000 such that the sum of their decimal digits is 12?

6. (10 points) The deck of cards contains 52 cards. There are 13 different kinds of cards: 2,3,4,5,6,7,8,9,10,J,D,K,A. There are also four suits: spades, clubs, hearts, and diamonds, each containing 13 cards, with one card of each kind in a suit. What is the probability that a given poker hand of five cards is a royal flush (A,K,D,J,10 of the same suit)?

7. (10 points) A fair coin is tossed five times. What is the probability of getting exactly four heads, given that at least one of the tosses is heads?

8. (20 points) Some tribe values boys so much that each of their families keeps making kids until they get a boy (after which they relax and make no more kids). On the other hand, no family can afford having more than five kids. So if the first five babies in a family are girls, the family stops making children anyway. Assuming that a boy and a girl are equally likely, consider two random variables X - “the number of boys in a family”, Y - “the number of girls in a family”.

- a) Find the expected values $E(X)$ and $E(Y)$ and compare them.
- b) Are X and Y independent?
- c) Find the expected value $E(X + Y)$.