CS 280, Prelim 2, November 9, 1999

Show all your work. Bare answers bring no credit. Do not simplify the answers.

1. (20 points)

a) Prove by induction that $7^n - 2^n$ is divisible by 5 for all positive integers n.

b) What is wrong with the following "proof"? Let $A(n)$ be a proposition "any n numbers are equal". We establish $A(n)$ for all positive n by induction.

BASIS STEP: $A(1)$ states that one number is equal to itself, which is obviously true.

INDUCTIVE STEP. Suppose $A(k)$, i.e. any k numbers are equal (the induction hypothesis). Take an arbitrary set X of $k+1$ numbers and remove one of them, say a. By the induction hypothesis, all numbers in the remaining set $X - \{a\}$ of k numbers are equal to each other. Remove another element b from X. By the induction hypothesis, all numbers from $X - \{b\}$ are also equal. Therefore all numbers from X are equal to a and to b.

2. (25 points)

a) A soccer team has 20 women in the roster, two of them are goalkeepers. One goalkeeper and 10 field players make a team. How many ways are there to combine a team from those in the roster?

b) Find the coefficient of x^4y^7 in $(3x - 2y)^{11}$.

c) How many ways are there to order letters in the word *NEVERTHELESS*?

d) How many solutions are there to the inequality $x + y \leq 19$ where x and y are nonnegative integers with $x \leq 15$ and $y \leq 15$?

3. (25 points) In bridge, the 52 cards are dealt to four players.

a) How many different ways are there to deal bridge hands to four players?

b) What is the probability that all four aces went to one hand?

c) Assume that the player number one got a hand without aces. What is the probability that there is a player that has a hand containing all four aces?

4. (30 points) A pair of fair dice is rolled.

a) What are the expected values of the minimum m and the maximum M of numbers on the dice?

b) Are the random variables m and M independent?

c) Find the expected value of $m + M$.

d) Find the expected value of $m \cdot M$.