CS280, Prelim 1 **March 6, 2001**

Full name:

Student ID:

Statement of integrity: I did not, and will not, break the rules of academic integrity on this exam.

(Signature)

Show all your work. Calculators are allowed.

1. (10 points) Find a proposition $A(p,q,r)$ given its truth table

p	q	r	$A(p,q,r)$
F	F	F	F
F	F	T	F
F	T	F	T
T	F	F	F
F	T	T	F
T	F	T	T
T	T	F	T
T	T	T	F

2. (10 points) Show that $\exists x(A(x) \rightarrow B(x))$ and $\exists x A(x) \rightarrow \exists x B(x)$ are not logically equivalent.

3. (10 points) Which of the following are true? (Yes or No answers, $P(X)$ is the powerset of X, i.e. the set of all subsets of X.)

(a) $\emptyset \in \{\{\emptyset\}\}$

(b) $\emptyset \subseteq \{\{\emptyset\}\}$

(c) $\{\emptyset\} \in \{\{\emptyset\}\}$

(d) $\{\emptyset\} \subseteq \{\{\emptyset\}\}$

(e) $\{\emptyset\} \in P(\{\{\emptyset\}\})$

4. (10 points) Let $f : S \longrightarrow T$ and $g : T \longrightarrow U$ be functions. The *composition* of functions g and f, denoted by $g \circ f$, is defined by $(g \circ f)(a) = g(f(a))$.
a) Prove that if $g \circ f$ is one-to-one, so is f.
b) Find an example where $g \circ f$ is one-to-one but g is not one-to-one.

5. (10 points) Let

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 0 & 1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} -6 & 5 \\ -4 & 3 \\ -2 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 5 & -6 \\ 3 & -4 \\ 1 & -2 \end{pmatrix}, \quad D = \begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{pmatrix}$$

Find $A \cdot B + A \cdot C + A \cdot D$.

6. (10 points) Fermat's Little Theorem: if p is a prime which does not divide a then

$$a^{p-1} \equiv 1 \pmod{p}$$

Find $7^{1000002}$ mod 101. The answer should be a nonnegative integer less than 101.

7. (20 points) Using the Chinese Remainder Theorem find a positive integer $x < 1287$ ($1287 = 9 \cdot 11 \cdot 13$) satisfying all three conditions below

$x \equiv 3(\mathrm{mod} 9)$,

$x \equiv 1(\mathrm{mod} 11)$,

$x \equiv 1(\mathrm{mod} 13)$.

8. (10 points) Prove by induction that for any positive integer n

$$\sum_{k=1}^{n} \frac{1}{(2k-1)(2k+1)} = \frac{n}{2n+1}$$