CS 280, Prelim 1, October 7, 1999

Show all your work.

1. (15 points) Determine which of the following propositions are tautologies

a) $(p \rightarrow \neg p) \leftrightarrow \neg p$ b) $(p \rightarrow \neg q) \leftrightarrow \neg(p \wedge q)$ c) $((\neg p \wedge q) \rightarrow r) \rightarrow ((\neg q \rightarrow p) \rightarrow r)$.

2. (15 points)
a) Establish the logical equivalence of $\neg\forall x(A \rightarrow B)$ and $\exists x(A \wedge \neg B)$.
b) Show that $\exists x(A(x) \wedge B(x))$ and $\exists x A(x) \wedge \exists x B(x)$ are not logically equivalent.

3. (15 points) The *composition* of functions f and g, denoted by $f \circ g$, is defined by $(f \circ g)(a) = f(g(a))$. The *inverse* of h is the function h^{-1} such that $h^{-1} \circ h$ and $h \circ h^{-1}$ are identity functions, i.e. $(h^{-1} \circ h)(a) = a$ and $(h \circ h^{-1})(b) = b$ for all a from the domain of h and all b from the codomain of h.
a) Give an example of f and g such that $f \circ g$ and $g \circ f$ are different
b) Suppose f and g are invertible. Show that $(f \circ g)^{-1}$ equals to $g^{-1} \circ f^{-1}$.

4. (10 points)
a) How many multiplications does the standard row-column algorithm uses to compute the product of an $m \times n$ matrix and an $n \times p$ matrix? Explain why.
b) Suppose you have to find $A \cdot B \cdot C$, were A is a 3×10 matrix, B - 10×50 matrix and C - 50×2 matrix. Which order should you choose: $(A \cdot B) \cdot C$ or $A \cdot (B \cdot C)$?

5. (15 points) Compute the greatest common divisor (gcd) of 156 and 93. Find integers x and y such that $156x + 93y = \gcd(156, 93)$.

6. (10 points)
a) Find the base 8 expansion of $(123)_{10}$.
b) Find the binary expansion of $(123)_{10}$

7. (20 points) By the Chinese Remainder Theorem for each integers a, b and c ($0 \leq a < 9$, $0 \leq b < 10$ and $0 \leq c < 11$) there is a unique nonnegative integer $x < 990 = 9 \cdot 10 \cdot 11$ such that $x \equiv a(\text{mod} 9)$, $x \equiv b(\text{mod} 10)$ and $x \equiv c(\text{mod} 11)$.

a) Find such a, b and c for $x = 801$.
b) Find a positive integer x satisfying $x \equiv 1(\text{mod} 9)$, $x \equiv 0(\text{mod} 10)$ and $x \equiv 1(\text{mod} 11)$.