CS 280, Final May 18, 2001

Full name:

Student ID:

Statement of integrity: I did not, and will not, break the rules of academic integrity on this exam.

(Signature)

Show all your work unless the problem states otherwise. Each problem is worth 10 points.

1. a) Determine which of the following propositions are tautologies. Show why.

a) $(p \to \neg q) \to (\neg q \to p)$ b) $[(p \lor q) \to r] \leftrightarrow [(p \to r) \land (q \to r)]$

b) Write down a proposition P over the variables p, q, r such that P is true only when p, q, r have truth values FFT or TFT.

c) Can such a P be constructed using the connectives $\land$ and $\to$ only (no truth constants for T and F are allowed)? If yes, do so. Otherwise show why not.

2. a) Let $L(x, y)$ be *x loves y.* Write the following as formal sentences with quantifiers

Everybody loves somebody

Everybody loves everybody

Somebody loves everyone

Somebody loves someone

b) Which of the following sentences are logically equivalent (answer only)? In principle you have to determine the equivalence of all 6 pairs of sentences.

$$1)\ \forall x(A(x) \to B), \quad 2)\ \forall x A(x) \to B, \quad 3)\ \exists x(A(x) \to B), \quad 4)\ \exists x A(x) \to B.$$

(B does not contain x.)

3. Which of the following are always true (A, B, C are arbitrary sets)? Yes-No answers.

if $A \subseteq B$ and $B \subseteq A$ then $A = B$

$\emptyset \in \{\emptyset\}$

$\emptyset \in A$

$\emptyset \subseteq A$

if $A \neq B$ and $B \neq C$ then $A \neq C$

$A - B = \overline{(\overline{A} - \overline{B})}$

$(A - B) \cap (B - A) = \emptyset$

$\emptyset \times A = \emptyset$

$A \times B = B \times A$

$\overline{A \cup B} = \overline{A} \cup \overline{B}$

4. How many one-to-one functions f from $\{1, 2, 3, 4, 5\}$ onto itself are there such that $f \circ f \circ f$ is the identity function?

5. Yes-No answers. Is it true that n^2 is $O(g(n))$, if $g(n)$ is

   n

   n^3

   $n \log n$

   2^n

   $n!$

   $n^2 \log n$

   $n^2 + \log n$

   $n^2 / \log n$

   $(n^4 + 1)/(n^2 + 1)$

   $(n^4 + 1)/(n + 1)$

6. Find 5^{20001} $(\mathbf{mod}\ 143)$. The answer should be a standard integer between 0 and 143.

7. a) Perform the following operations on binary numbers:

$1101 + 1011 =$

$1101 \cdot 1011 =$

$1101/1011 =$ (find a quotient and a remainder)

b) Transform $(1101)_2$ into decimal.

c) Transform $(1101)_{10}$ into binary.

8. a) Give an example of matrices A and B such that $AB \neq BA$.

b) Find all 2×2 zero-one matrices whose boolean square is the zero matrix. The answer only.

c) How many integer additions and how many integer multiplications does it take to multiply two 5×5 integer matrices using the standard row-column algorithm (answer only)?

9. a) Suppose that $S(n)$ is a proposition involving a nonnegative integer n, and suppose that if $S(k)$ is true then so is $S(k+2)$. Which of the following are possible (answer only)?

$S(n)$ holds for all $n \geq 0$,

$S(n)$ holds for all $n \geq 1$, but $S(0)$ is false,

$S(n)$ is false for all $n \geq 0$,

$S(n)$ is true for all $n \leq 100$ and false for all $n > 100$,

$S(n)$ is false for all $n \leq 100$ and true for all $n > 100$.

b) A collection S of strings of characters is defined recursively by

i) the empty string is in S, ii) if X belongs to S then so does aXb.

Which of the following belong to S (Yes-No answers):

a

b

c

ab

abb

$aabb$

10. a) A questionnaire is sent to 13 freshmen, 5 sophomores, 15 juniors, and 20 seniors. A student won't necessarily return his/her questionnaire. How many questionnaires must be received to ensure getting 9 from the same class?

b) How many bit strings are there of length 10 with two or more 1's in the string?

11. a) Find the number of positive integer solutions of $x + y + z \leq 100$.

b) A hostess wishes to invite 6 dinner guests. In how many ways can she place them on 6 distinguished seats?

c) The same question for a round table with indistinguishable seats.

12. Two fair dice are rolled. What is the probability that

a) the number on the first die is strictly less than the number on the second die?

b) one of those numbers is strictly less than the other one?

c) the product of those numbers is even given that their sum equals 6 ?

13. A couple agrees to keep having children until they have at least one boy and one girl, but not more than 5 children. Assume that boys and girls are equiprobable and that the births are mutually independent. What is the expected value and variance of the number of

a) children ?

b) girls ?

14. An integer is randomly selected from 1 to 1000. What is the probability that it is divisible by 2 or by 3 or by 5?

15. Give an example of a relation R that is

a) symmetric, not reflexive

b) reflexive, symmetric, not transitive.

16. a) How many vertices and edges does the graph Q_5 have?

b) A graph has 7 vertices, 3 of them of degree two and 4 of degree one. Is this graph connected? Why?

17. Which of the following graphs have an Euler circuit? An Euler path? (Answer only)

K_5

Q_3

Q_4

$K_{2,5}$

$K_{3,5}$

18. Which of the following graphs are planar? (Answer only)

K_5

Q_3

$K_{2,5}$

$K_{3,5}$

$K_{4,5}$