Priority Queues and Heaps

Lecture 25
CS2112 Fall 2012

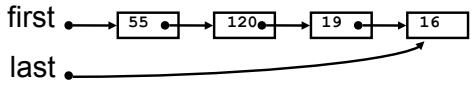
The Bag Interface

```
interface Bag<E> {
   void insert(E obj);
   E extract(); //extract some element
   boolean isEmpty();
}
```

Examples: Stack, Queue, PriorityQueue

Stacks and Queues as Lists

- Stack (LIFO) implemented as list
  - `insert()`, `extract()` from front of list
- Queue (FIFO) implemented as list
  - `insert()` on back of list, `extract()` from front of list
- All `Bag` operations are O(1)



Priority Queue

- A `Bag` in which data items are `Comparable`
- *lesser* elements (as determined by `compareTo()`) have *higher* priority
- `extract()` returns the element with the highest priority = least in the `compareTo()` ordering
- break ties arbitrarily

Priority Queue Examples

- Scheduling jobs to run on a computer
  - default priority = arrival time
  - priority can be changed by operator
- Scheduling events to be processed by an event handler
  - priority = time of occurrence
- Airline check-in
  - first class, business class, coach
  - FIFO within each class

PQ Application: Simulation

- Example: Probabilistic model of bank-customer arrival times and transaction times, how many tellers are needed?
  - Assume we have a way to generate random inter-arrival times
  - Assume we have a way to generate transaction times
  - Can simulate the bank to get some idea of how long customers must wait

Time-Driven Simulation

- Check at each *tick* to see if any event occurs

Event-Driven Simulation

- Advance clock to next event, skipping intervening *ticks*
- This uses a PQ!

`java.util.PriorityQueue<E>`

```
boolean add(E e) {...} //insert an element (insert)
void clear() {...} //remove all elements
E peek() {...} //return min element without removing
               //(null if empty)
E poll() {...} //remove min element (extract)
                  //(null if empty)
int size() {...}
```

Priority Queues as Lists

- Maintain as unordered list
  - **`insert()`** puts new element at front – O(1)
  - **`extract()`** must search the list – O(n)
- Maintain as ordered list
  - **`insert()`** must search the list – O(n)
  - **`extract()`** gets element at front – O(1)
- In either case, $O(n^2)$ to process n elements

Can we do better?

Important Special Case

- Fixed number of priority levels 0, ..., p – 1
- FIFO within each level
- Example: airline check-in

- `insert()` – insert in appropriate queue – O(1)
- `extract()` – must find a nonempty queue – O(p)

Heaps

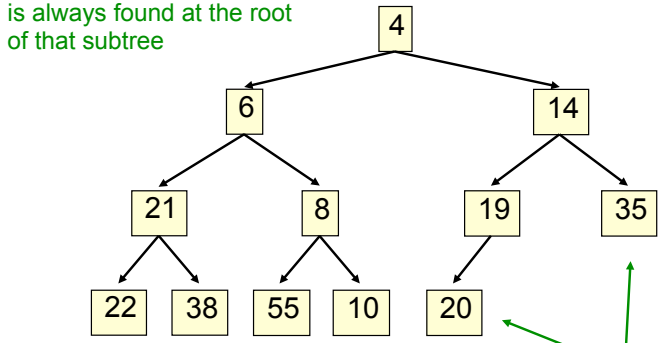
- A *heap* is a concrete data structure that can be used to implement priority queues
- Gives better complexity than either ordered or unordered list implementation:
  - `insert()`: O(log n)
  - `extract()`: O(log n)
- O(n log n) to process n elements
- Do not confuse with *heap memory*, where the Java virtual machine allocates space for objects – different usage of the word *heap*

Heaps

- Binary tree with data at each node
- Satisfies the *Heap Order Invariant*:

The least (highest priority) element of any subtree is found at the root of that subtree

Least element in any subtree is always found at the root of that subtree



But it is possible to have smaller elements deeper in the tree!

Examples of Heaps

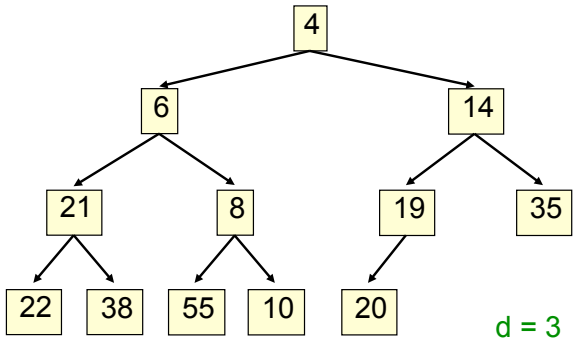
- Ages of people in family tree
  - parent is always older than children, but you can have an uncle who is younger than you
- Salaries of employees of a company
  - bosses generally make more than subordinates, but a VP in one subdivision may make less than a Project Supervisor in a different subdivision

Balanced Heaps

Two restrictions:

1. Any node of depth < d – 1 has exactly 2 children, where d is the height of the tree
   - implies that any two maximal paths (path from a root to a leaf) are of length d or d – 1, and the tree has at least 2^d nodes
1. All maximal paths of length d are to the left of those of length d – 1

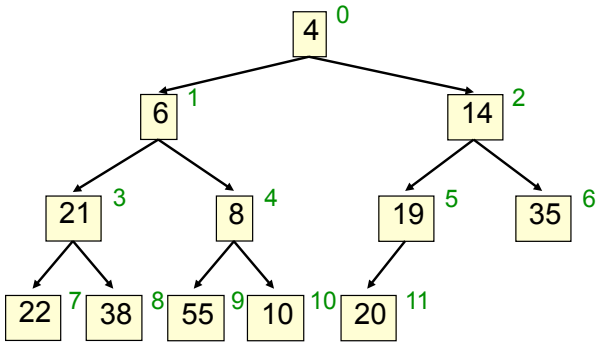
A Balanced Heap



Store in an ArrayList

- Elements of the heap are stored in the array in order, going across each level from left to right, top to bottom
- The children of the node at array index n are found at 2n + 1 and 2n + 2
- The parent of node n is found at (n – 1)/2

Store in an ArrayList or Vector

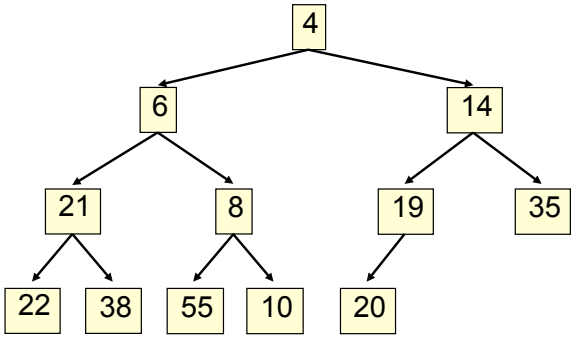


children of node n are found at 2n + 1 and 2n + 2

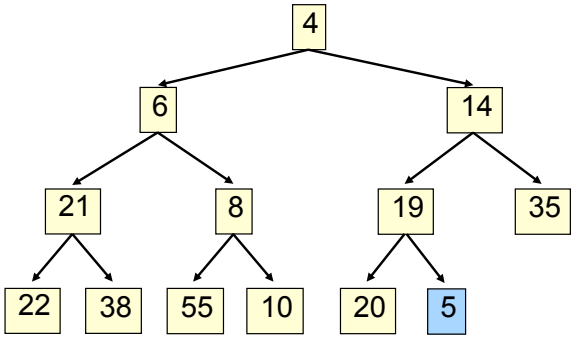
insert()

- Put the new element at the end of the array
- If this violates heap order because it is smaller than its parent, swap it with its parent
- Continue swapping it up until it finds its rightful place
- The heap invariant is maintained!

insert()



insert()



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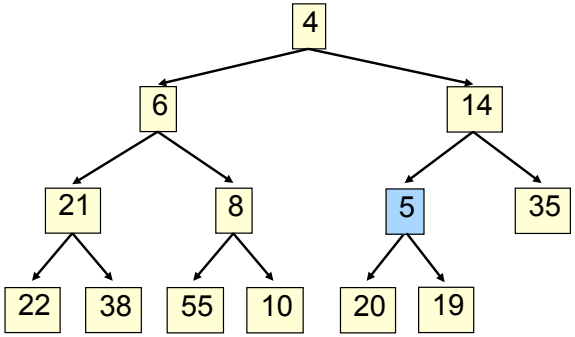
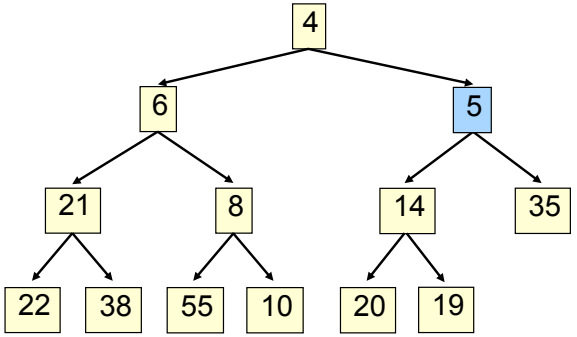


insert()



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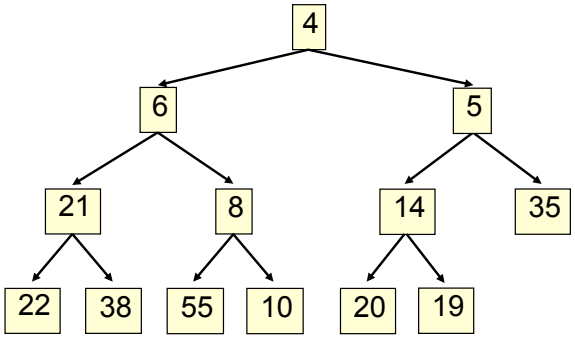

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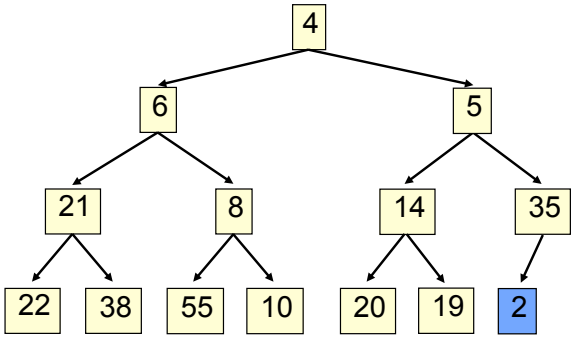

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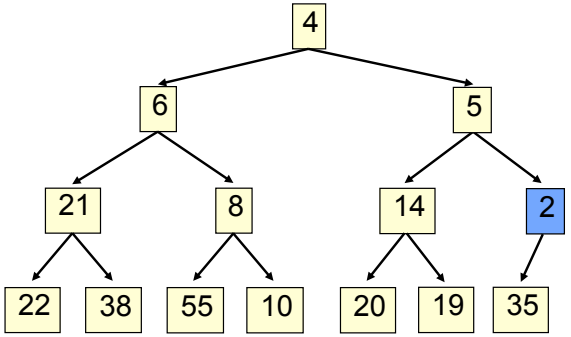

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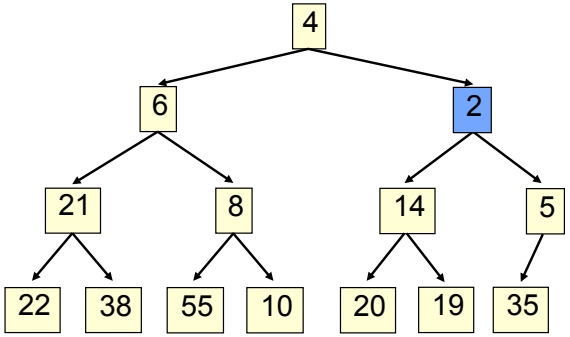

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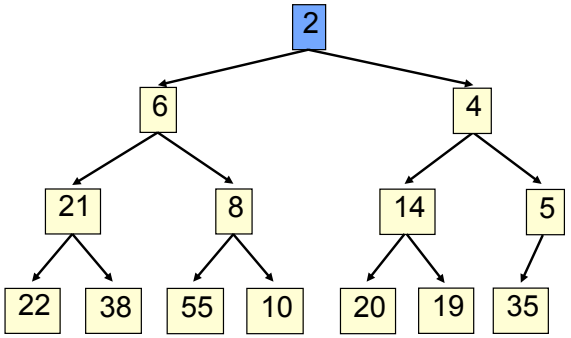

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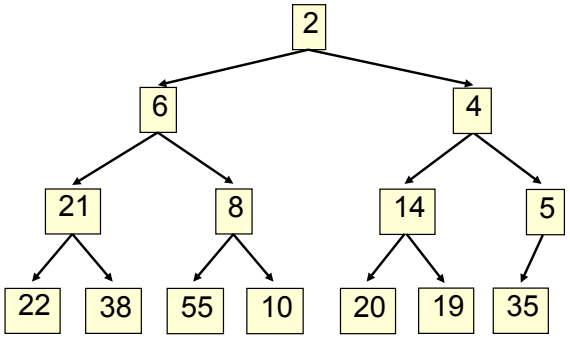


insert()

- Time is O(log n), since the tree is balanced
  - size of tree is exponential as a function of depth
  - depth of tree is logarithmic as a function of size

insert()

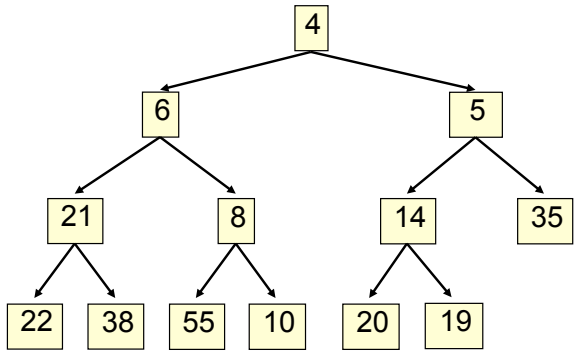
```
class PriorityQueue<E extends Comparable<E>>
                                        extends ArrayList<E> {
   public void put(E obj) {
      add(obj); //add new element to end of array
      rotateUp(size() - 1);
   }

 private void rotateUp(int index) {
      if (index == 0) return;
      int parent = (index - 1)/2;
      if (get(parent).compareTo(get(index)) <= 0) return;
      swap(index, parent);
      rotateUp(parent);
   }
}
```

extract()

- Remove the least element – it is at the root
- This leaves a hole at the root – fill it in with the last element of the array
- If this violates heap order because the root element is too big, swap it down with the smaller of its children
- Continue swapping it down until it finds its rightful place
- The heap invariant is maintained!

extract()



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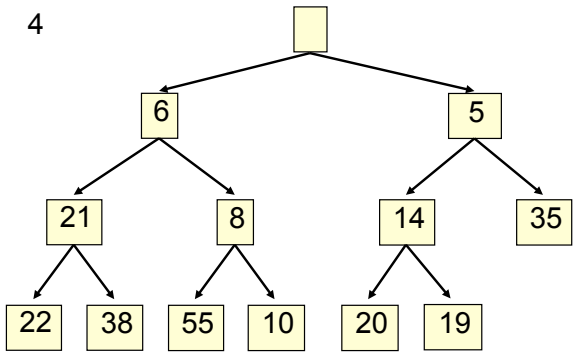

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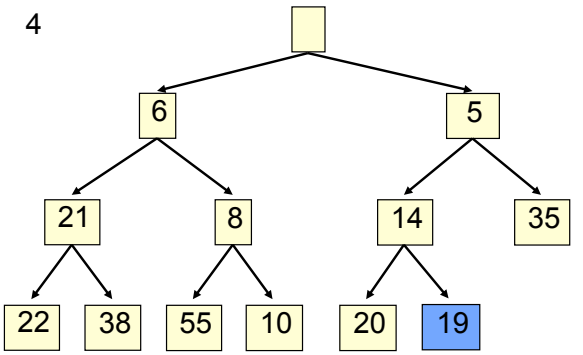

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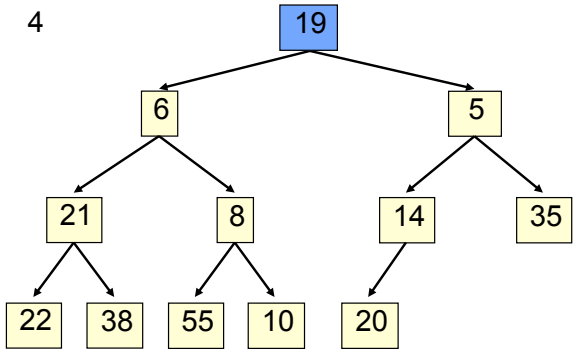

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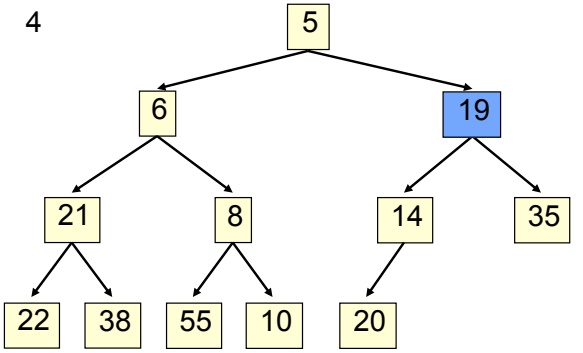

extract()

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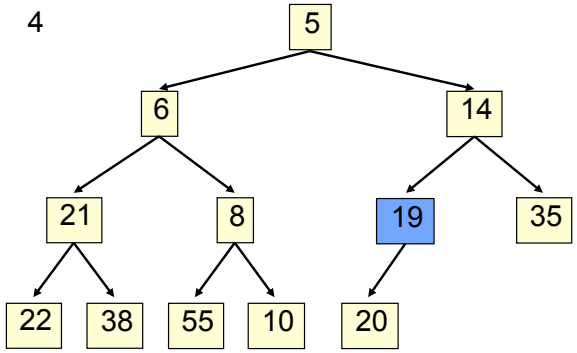

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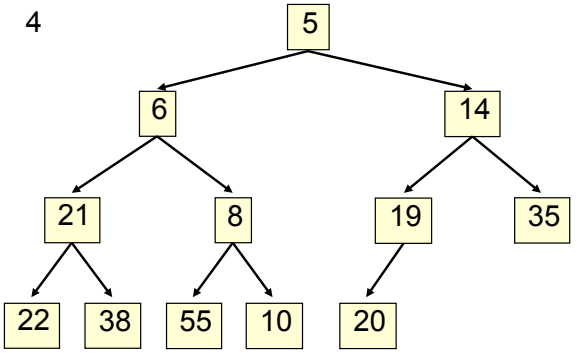

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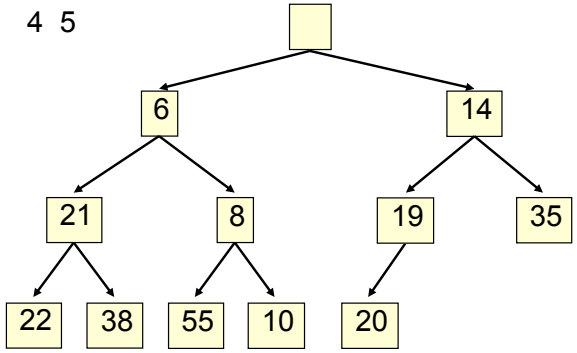

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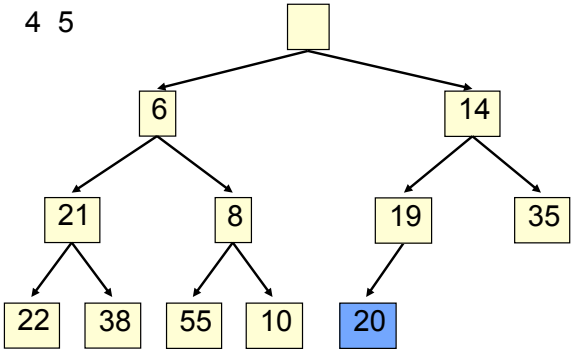

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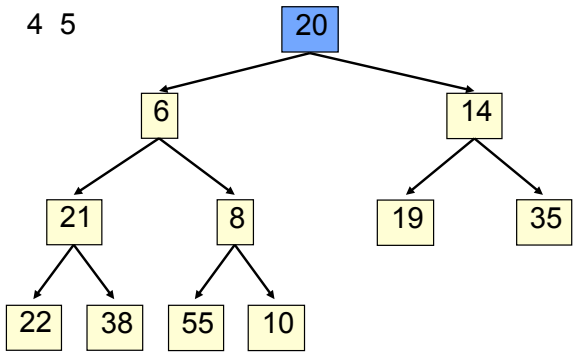

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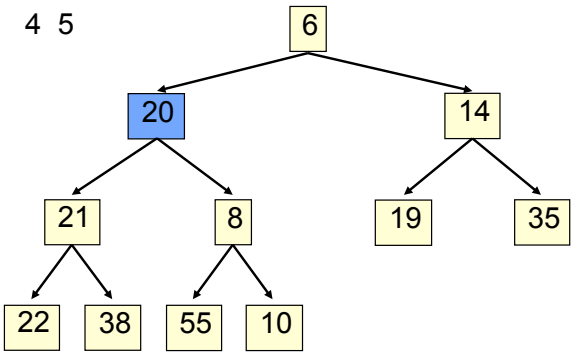

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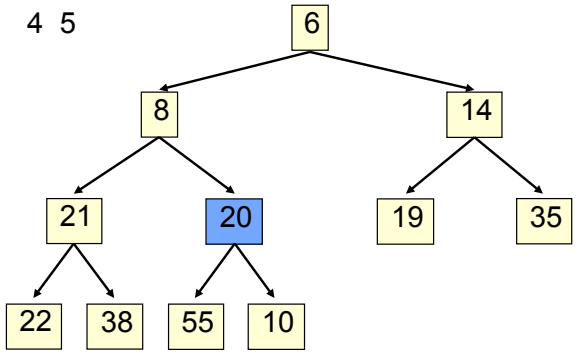

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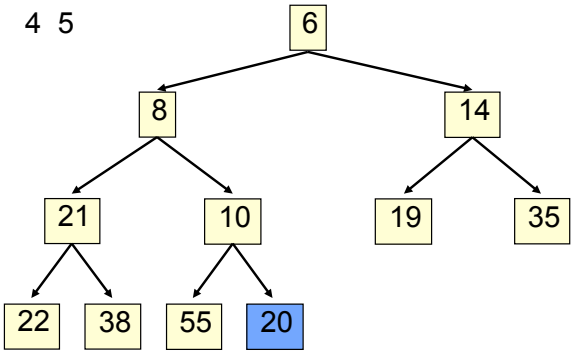

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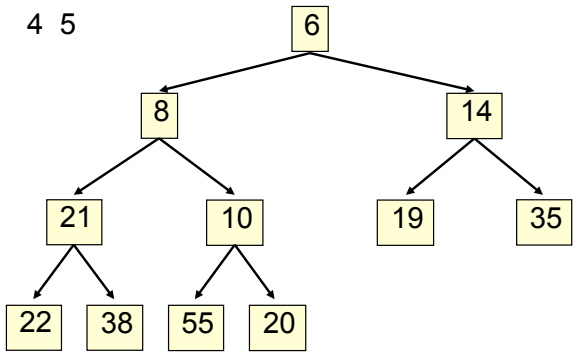

extract()

- Time is O(log n), since the tree is balanced

extract()

```
public E get() {
   if (isEmpty()) throw new NoSuchElementException();
   E temp = get(0);
   set(0, get(size() - 1));
   remove(size() - 1);
   rotateDown(0);
   return temp;
}
private void rotateDown(int index) {
   int child = 2*(index + 1); //right child
   if (child >= size()
      || get(child - 1).compareTo(get(child)) < 0)
         child -= 1;
   if (child >= size()) return;
   if (get(index).compareTo(get(child)) <= 0) return;
   swap(index, child);
   rotateDown(child);
}
```

HeapSort

Given a **`Comparable`** array of length n,

- Put all n elements into a heap– O(n log n)
- Repeatedly get the min – O(n log n)

```
public static <E extends Comparable<E>> void heapSort(E[] a) {
   PriorityQueue<E> pq = new PriorityQueue<E>();
   for (E x : a) pq.put(x);
   for (int i = 0; i < a.length; i++) a[i] = pq.get();
}
```

- Can construct heap faster, in O(n) time:
  1. Append all elements to end of array
  2. Establish heap order invariant by walking array backward from index n/2 and establishing invariant for subtree of each element.