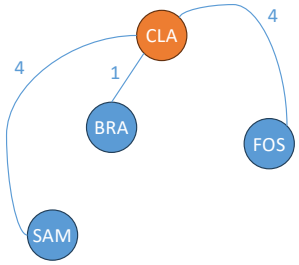
Weighted shortest paths example

Best known path

Vertex	Distance

Best possible path

Vertex	Distance
CLA	0



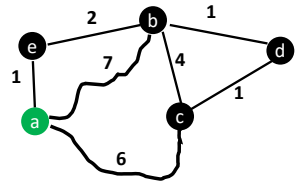
Invariant and theorem

- "Settled set" – vertices whose neighbors have all been discovered
  - Path distance is shortest possible
  - Neighbors are either in the frontier or are settled themselves
- For each frontier vertex, we know the shortest path so far, and it only goes through settled vertices
  - Paths are only updated when v is being visited, about to be settled

- Theorem:
  - If *f* is the vertex in the frontier with the *smallest* candidate path, then that path is the shortest possible path from *start* to *f*
    - Consequence: *f* is ready to be settled

Exercise: Tracing Dijkstra's algorithm

```
start.discovered = true;
start.dist = 0;
frontier.add(start, start.dist);
while (!frontier.isEmpty()) {
  Vertex v = frontier.removeMin();
  for (Edge e : v.outgoing) {
    Vertex neighbor = e.to;
    double dist = v.dist + e.weight;
    if (!neighbor.discovered || dist < neighbor.dist) {
      neighbor.discovered = true;
      neighbor.dist = dist;
      frontier.addOrUpdate(neighbor, dist);
    }
  }
}
```



Frontier (priority queue)					
a					

Vertex state

Vertex	a	b	c	d	e
Dist	0				
Pred	null				

Exercise: Complexity analysis

Self cost Total cost

```
while (!frontier.isEmpty()) {
  Vertex v = frontier.removeMin();
  for (Edge e : v.outgoing) {
    Vertex neighbor = e.to;
    double dist = v.dist + e.weight;
    if (dist < neighbor.dist) {
      neighbor.dist = dist;
      frontier.addOrUpdate(neighbor,
          dist);
    }
  }
}
```