

PRIORITY QUEUES AND HEAPS

Lecture 17
CS2110 Fall 2010

The Bag Interface

2

□ A Bag:

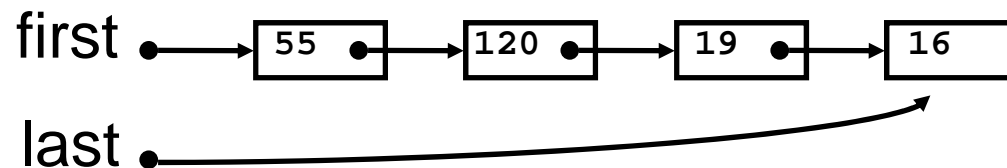
```
interface Bag<E> {  
    void insert(E obj);  
    E extract(); //extract some element  
    boolean isEmpty();  
}
```

Examples: Stack, Queue, PriorityQueue

Stacks and Queues as Lists

3

- Stack (LIFO) implemented as list
 - `insert()`, `extract()` from front of list
- Queue (FIFO) implemented as list
 - `insert()` on back of list, `extract()` from front of list
- All **Bag** operations are $O(1)$



Priority Queue

4

- A **Bag** in which data items are **Comparable**
- *lesser* elements (as determined by **compareTo()**) have *higher* priority
- **extract()** returns the element with the highest priority = least in the **compareTo()** ordering
- break ties arbitrarily

Priority Queue Examples

5

- Scheduling jobs to run on a computer
 - default priority = arrival time
 - priority can be changed by operator
- Scheduling events to be processed by an event handler
 - priority = time of occurrence
- Airline check-in
 - first class, business class, coach
 - FIFO within each class

java.util.PriorityQueue<E>

6

```
boolean add(E e) {...} //insert an element (insert)
void clear() {...} //remove all elements
E peek() {...} //return min element without removing
                //(null if empty)
E poll() {...} //remove min element (extract)
                //(null if empty)
int size() {...}
```

Priority Queues as Lists

7

- Maintain as **unordered list**
 - **insert ()** puts new element at front – $O(1)$
 - **extract ()** must search the list – $O(n)$
- Maintain as **ordered list**
 - **insert ()** must search the list – $O(n)$
 - **extract ()** gets element at front – $O(1)$
- In either case, $O(n^2)$ to process n elements

Can we do better?

Important Special Case

8

- Fixed number of priority levels $0, \dots, p - 1$
- FIFO within each level
- Example: airline check-in

- **insert ()** – insert in appropriate queue – $O(1)$
- **extract ()** – must find a nonempty queue – $O(p)$

Heaps

9

- A *heap* is a concrete data structure that can be used to implement priority queues
- Gives better complexity than either ordered or unordered list implementation:
 - `insert()`: $O(\log n)$
 - `extract()`: $O(\log n)$
- $O(n \log n)$ to process n elements
- Do not confuse with *heap memory*, where the Java virtual machine allocates space for objects – different usage of the word *heap*

Heaps

10

- Binary tree with data at each node
- Satisfies the *Heap Order Invariant*:

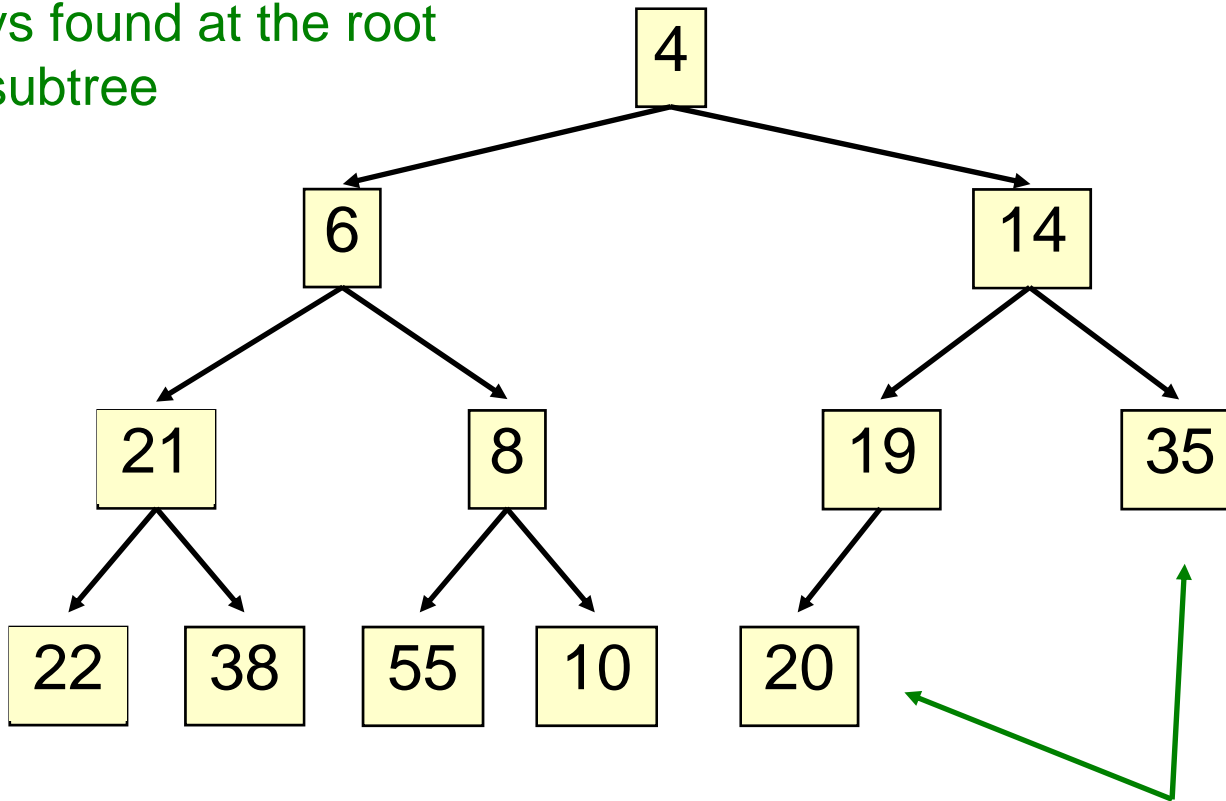
The least (highest priority) element of any subtree is found at the root of that subtree

- Size of the heap is “fixed” at n . (But can usually double n if heap fills up)

Heaps

11

Least element in any subtree
is always found at the root
of that subtree



Note: $19, 20 < 35$: we can often find smaller elements deeper in the tree!

Examples of Heaps

12

- Ages of people in family tree
 - parent is always older than children, but you can have an uncle who is younger than you
- Salaries of employees of a company
 - bosses generally make more than subordinates, but a VP in one subdivision may make less than a Project Supervisor in a different subdivision

Balanced Heaps

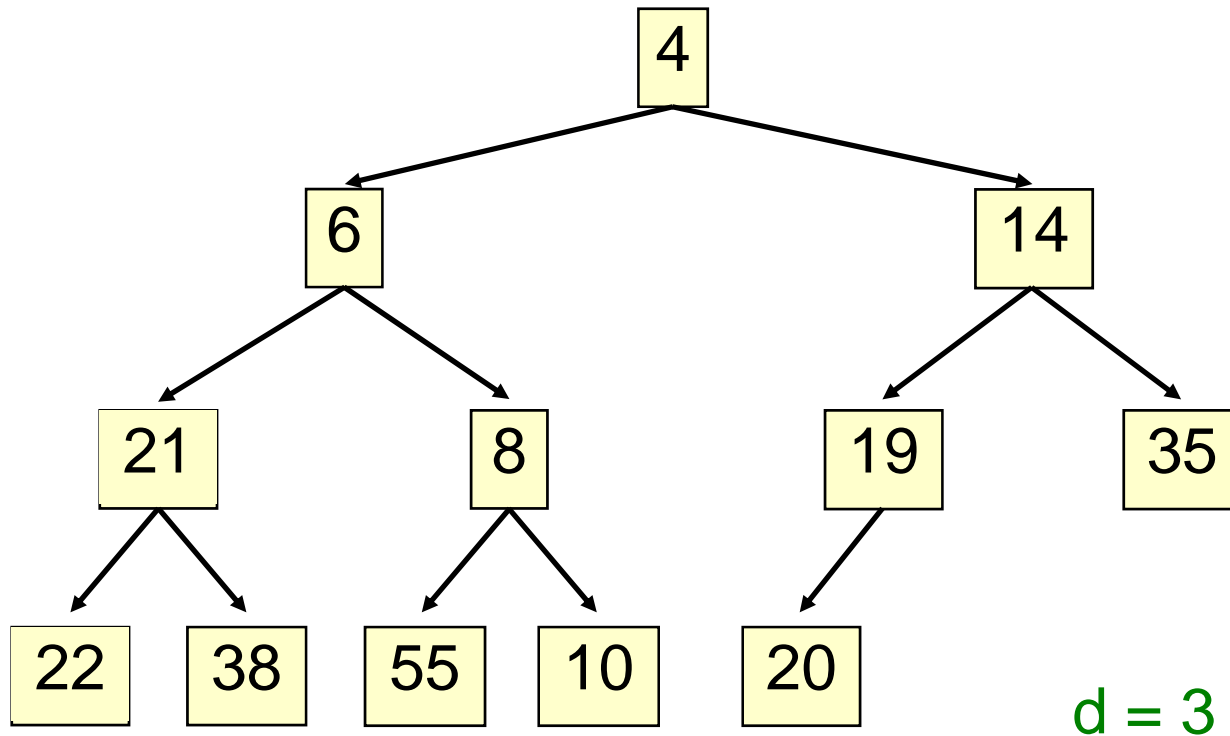
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These add two restrictions:

1. Any node of depth $< d - 1$ has exactly 2 children, where d is the height of the tree
 - implies that any two maximal paths (path from a root to a leaf) are of length d or $d - 1$, and the tree has at least 2^d nodes
- All maximal paths of length d are to the left of those of length $d - 1$

Example of a Balanced Heap

14



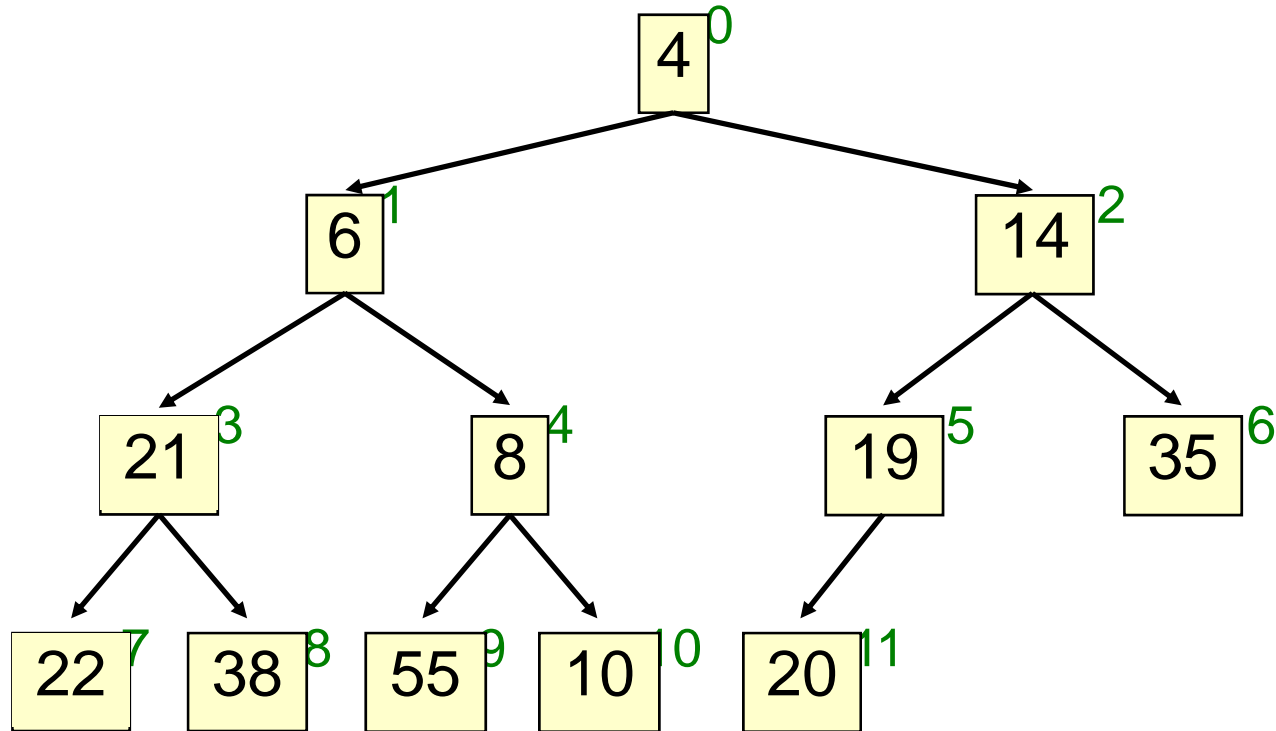
Store in an ArrayList or Vector

15

- Elements of the heap are stored in the array in order, going across each level from left to right, top to bottom
- The children of the node at array index n are found at $2n + 1$ and $2n + 2$
- The parent of node n is found at $(n - 1)/2$

Store in an ArrayList or Vector

16



children of node n are found at $2n + 1$ and $2n + 2$

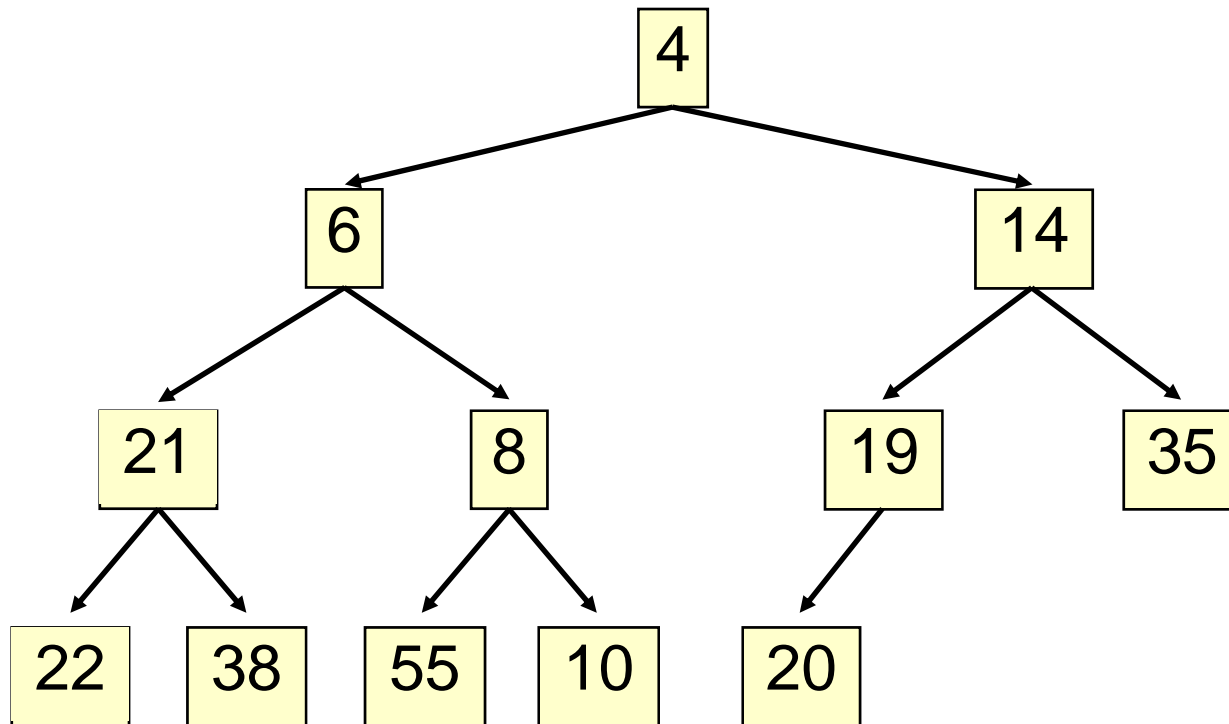
insert ()

17

- Put the new element at the end of the array
- If this violates heap order because it is smaller than its parent, swap it with its parent
- Continue swapping it up until it finds its rightful place
- The heap invariant is maintained!

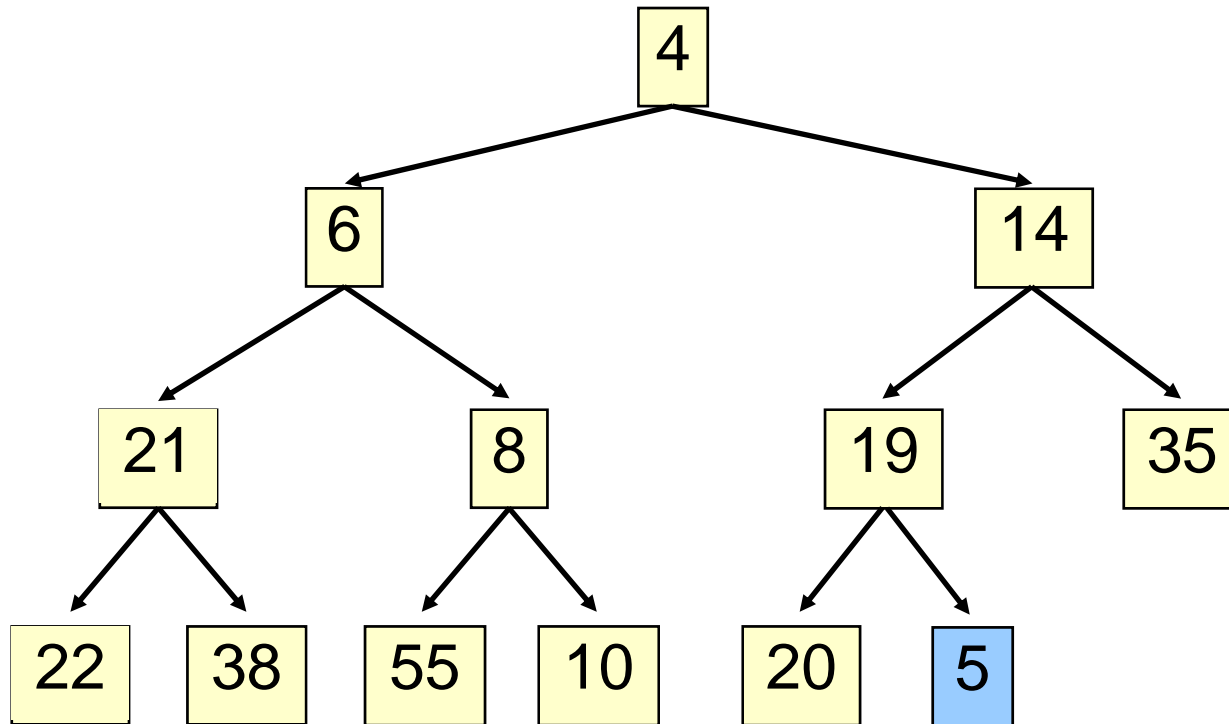
insert()

18



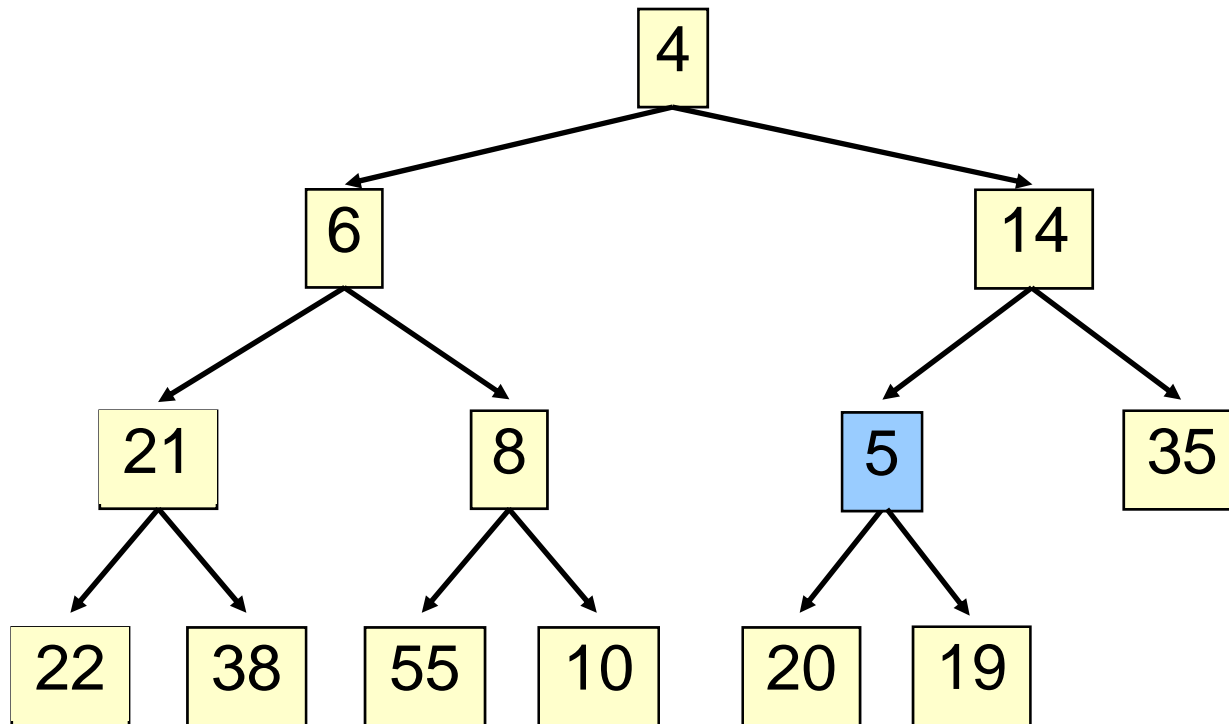
insert()

19



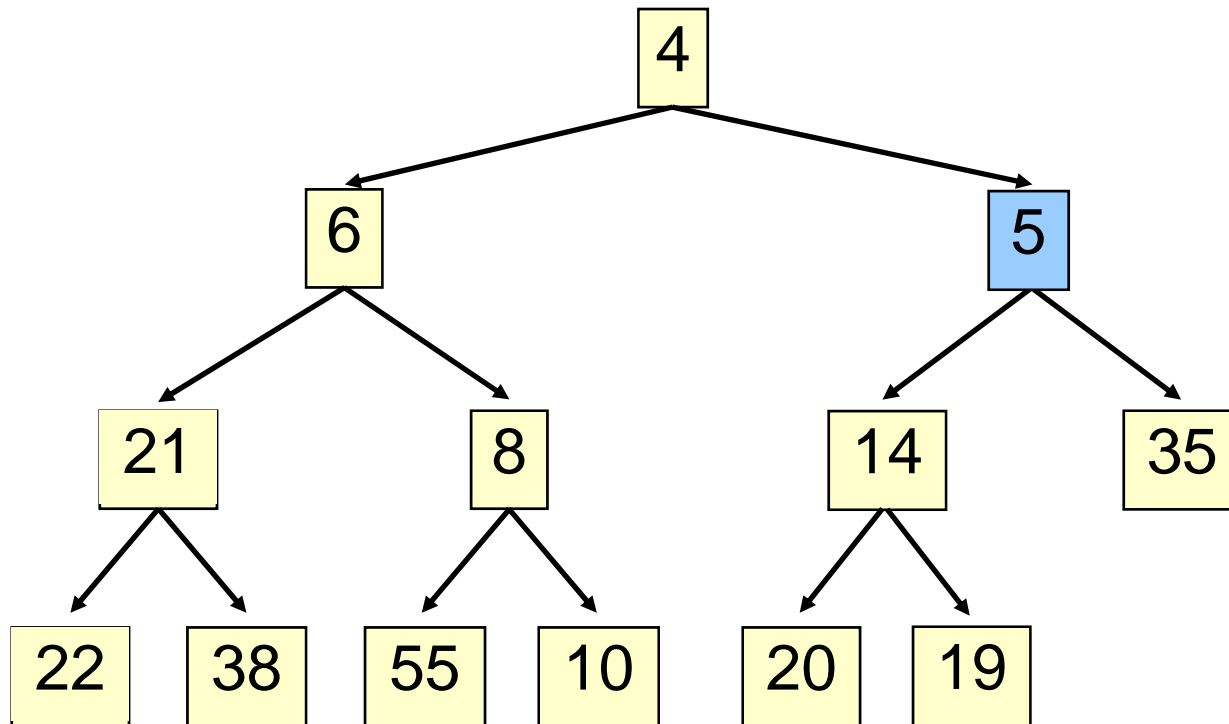
insert()

20



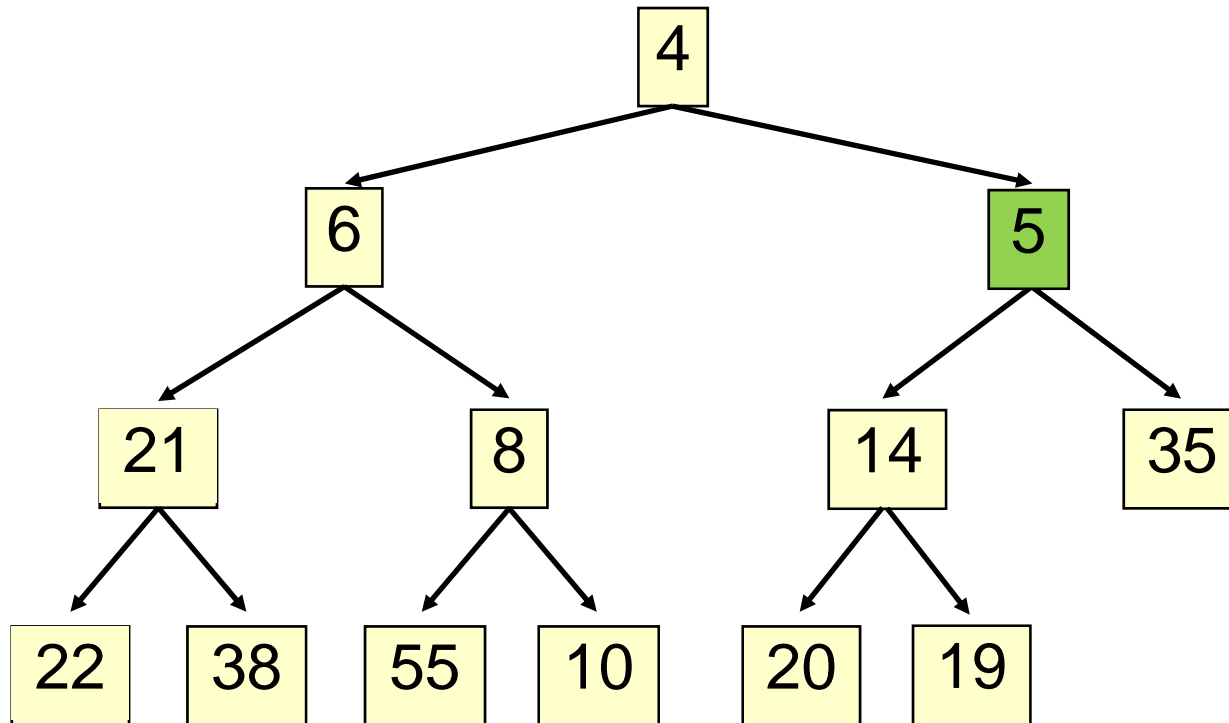
insert()

21

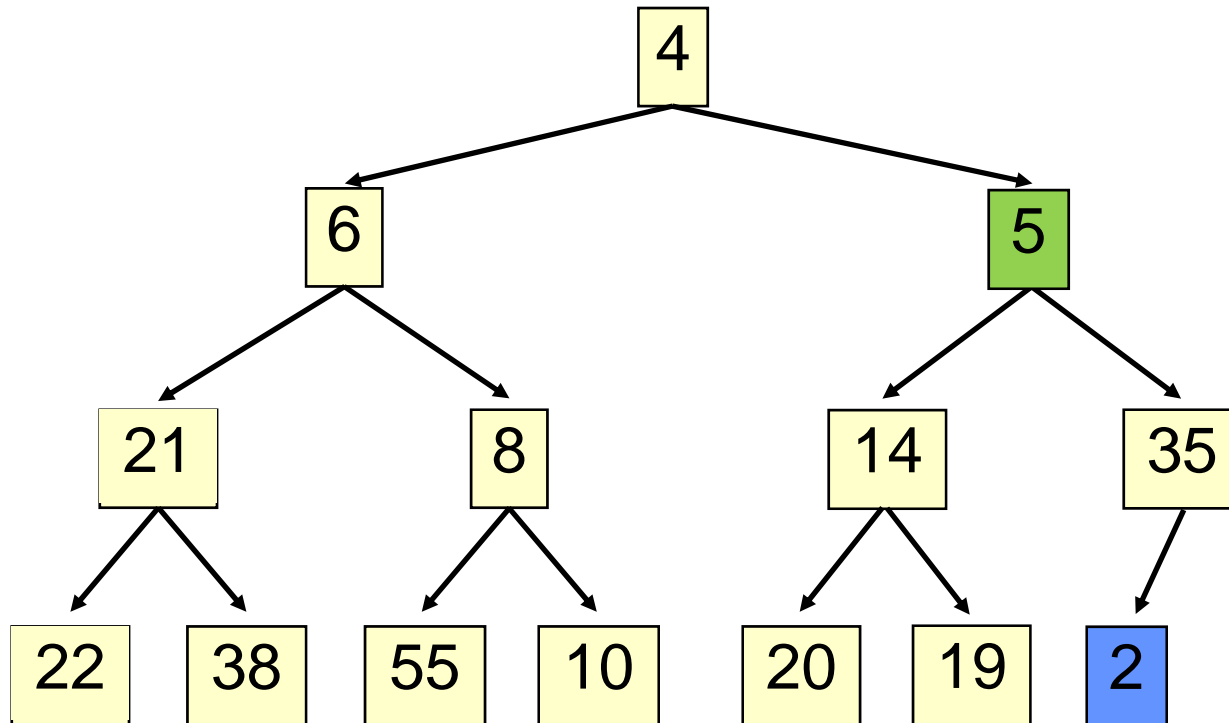


insert()

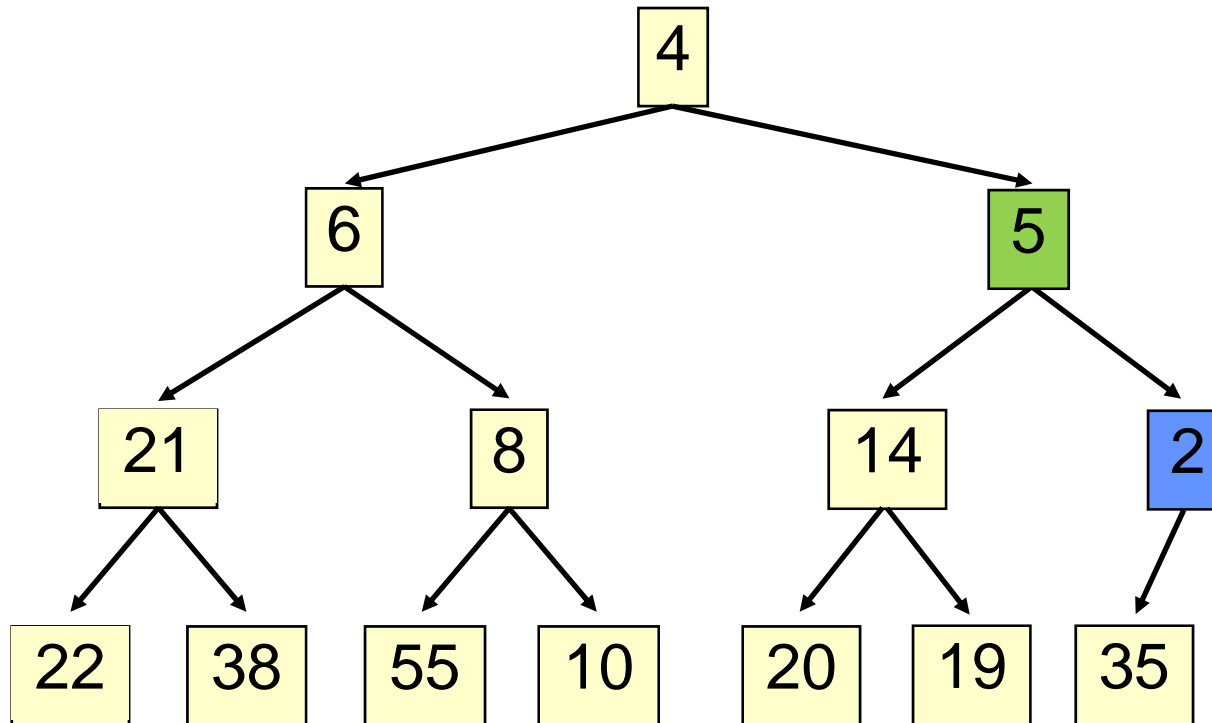
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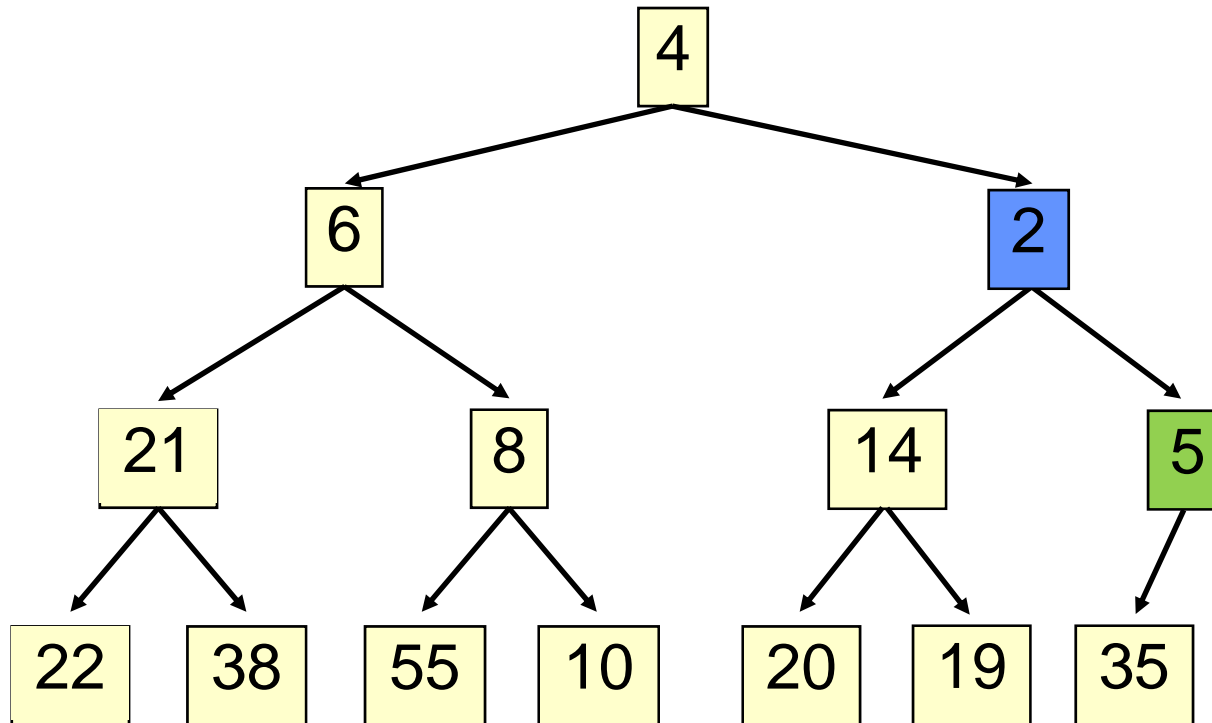
insert()



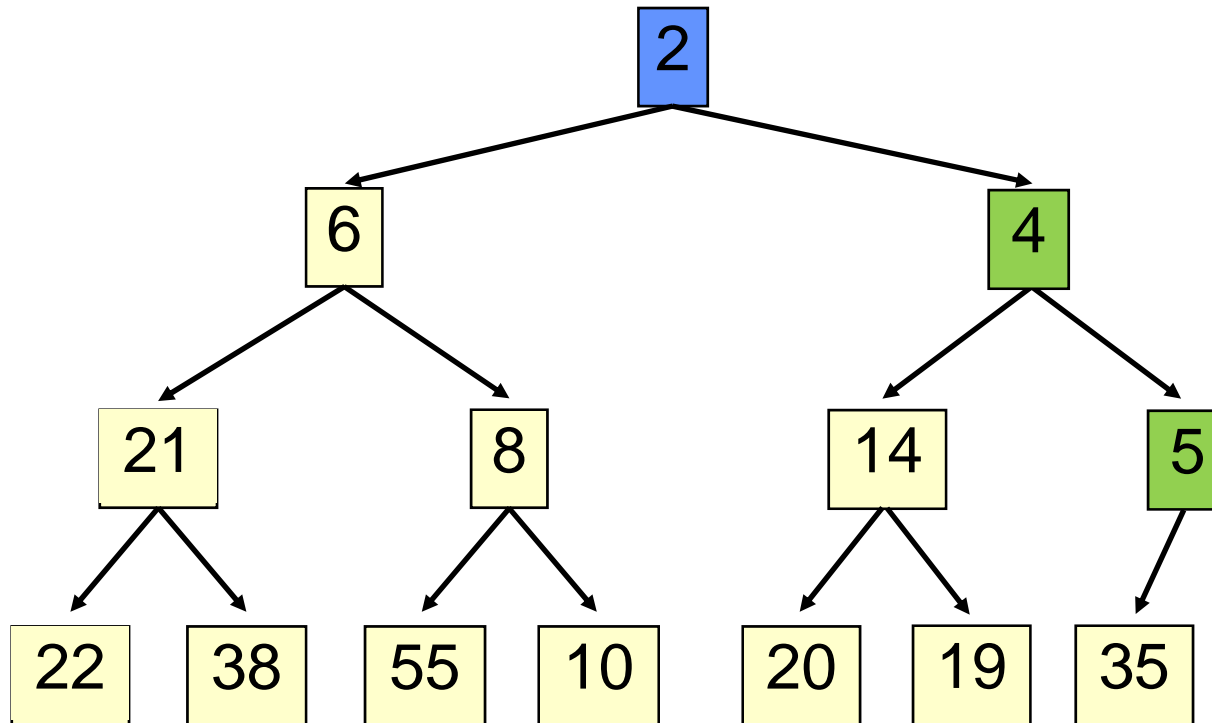
insert()



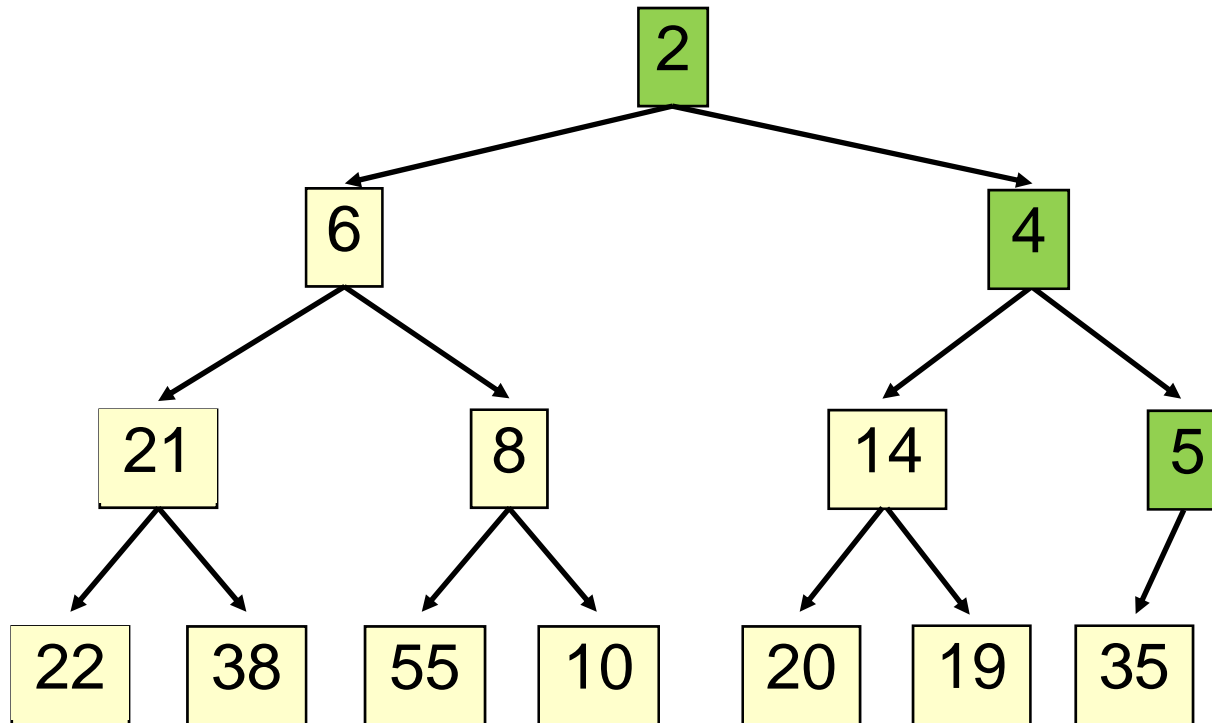
insert()



insert()



insert()



insert ()

28

- Time is $O(\log n)$, since the tree is balanced
 - size of tree is exponential as a function of depth
 - depth of tree is logarithmic as a function of size

insert()

insert()

29

```
class PriorityQueue<E> extends java.util.Vector<E> {  
  
    public void insert(E obj) {  
        super.add(obj); //add new element to end of array  
        rotateUp(size() - 1);  
    }  
  
    private void rotateUp(int index) {  
        if (index == 0) return;  
        int parent = (index - 1)/2;  
        if (elementAt(parent).compareTo(elementAt(index)) <= 0)  
            return;  
        swap(index, parent);  
        rotateUp(parent);  
    }  
}
```

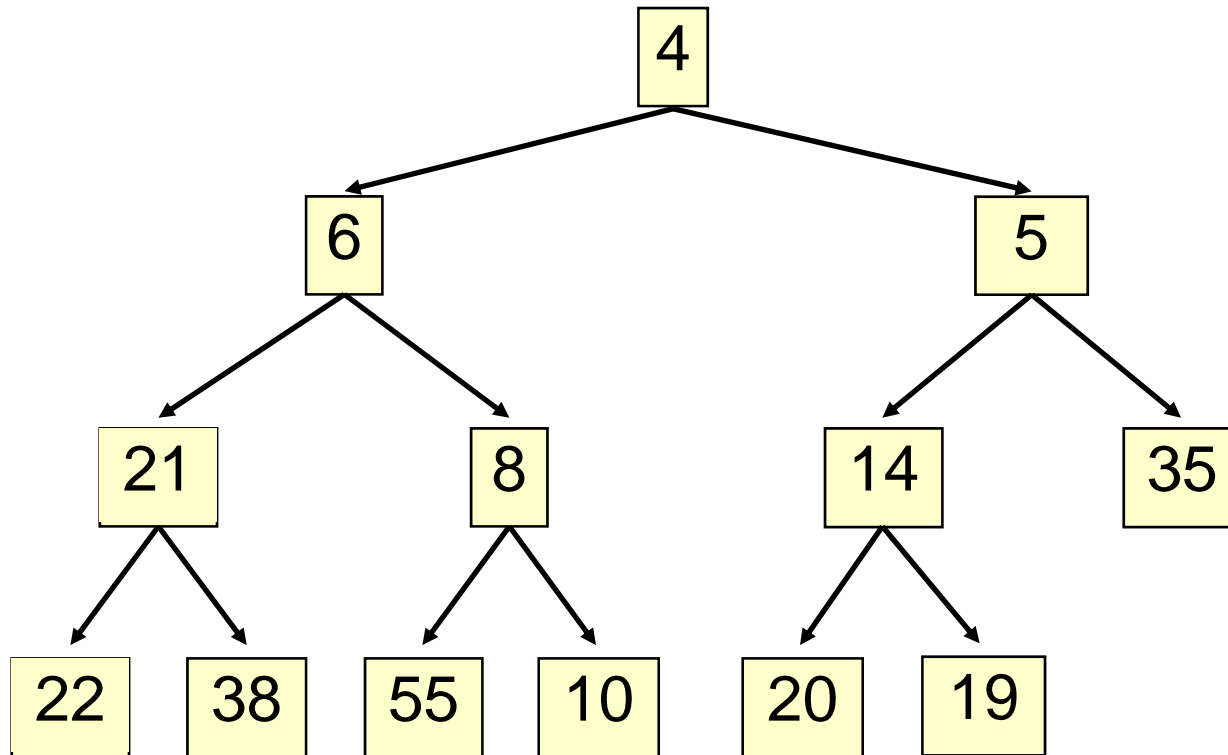
extract ()

30

- Remove the least element – it is at the root
- This leaves a hole at the root – fill it in with the last element of the array
- If this violates heap order because the root element is too big, swap it down with the smaller of its children
- Continue swapping it down until it finds its rightful place
- The heap invariant is maintained!

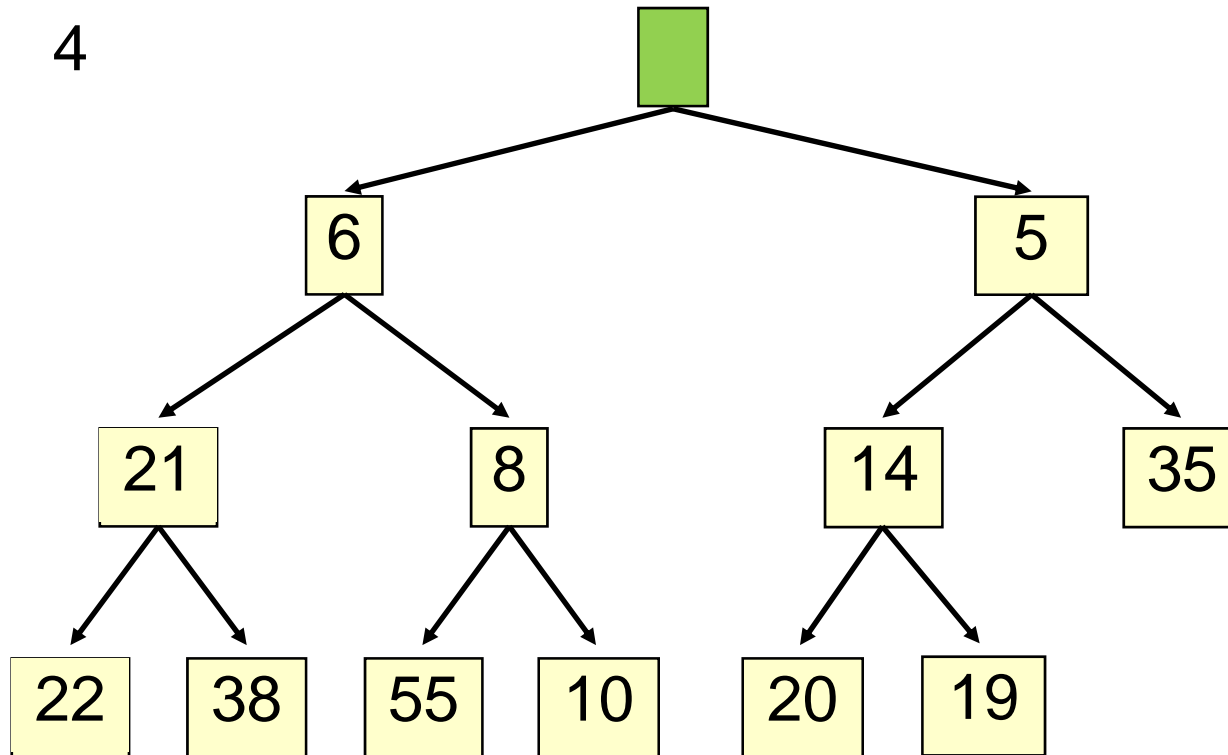
extract ()

31



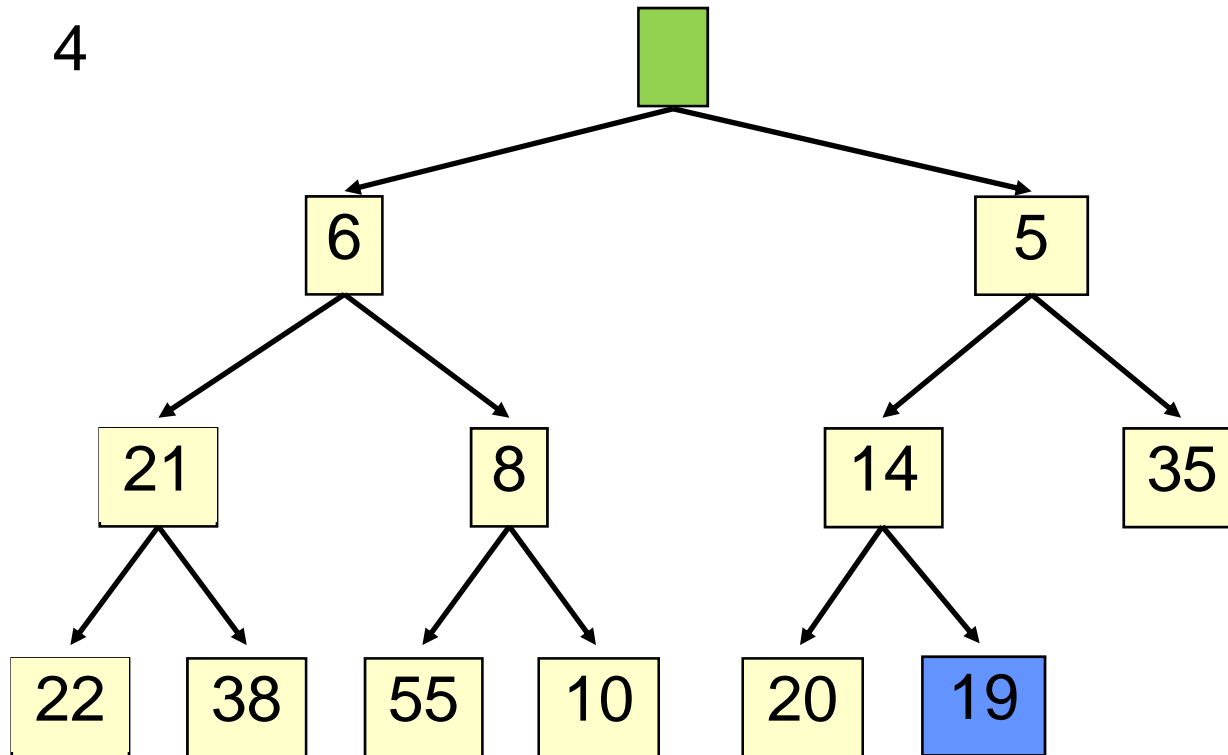
extract()

32



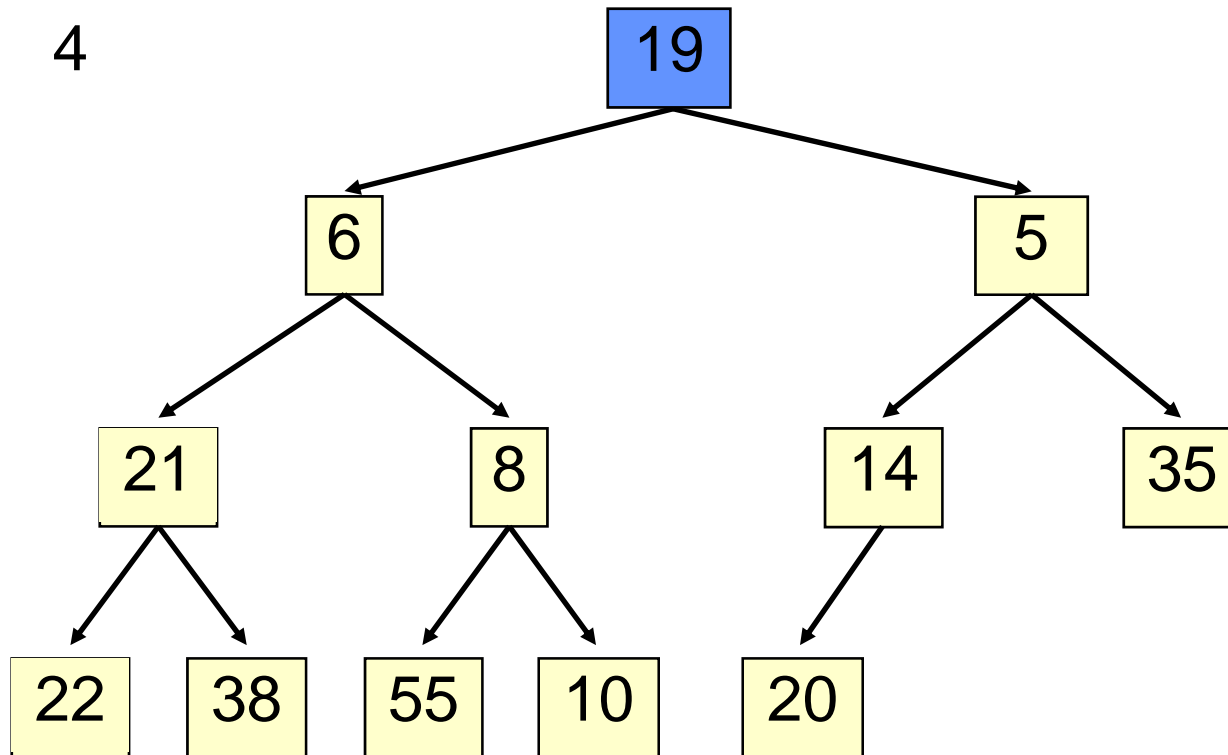
extract ()

33



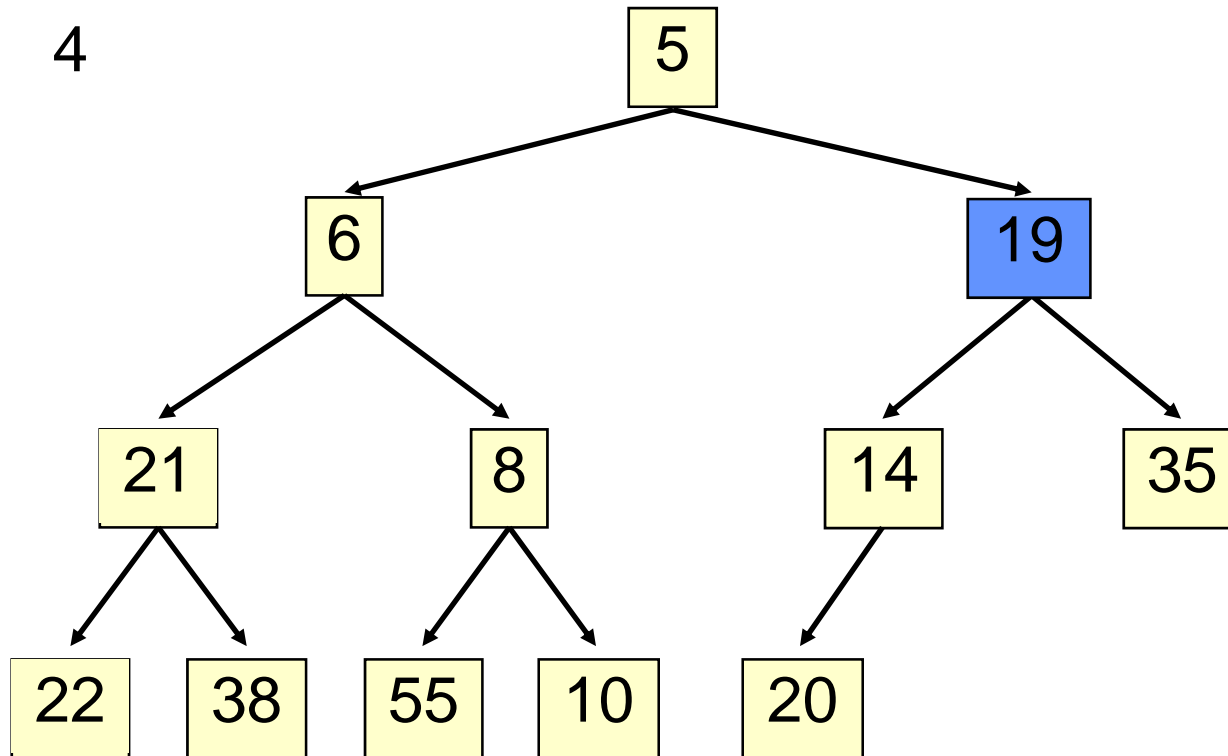
extract ()

34



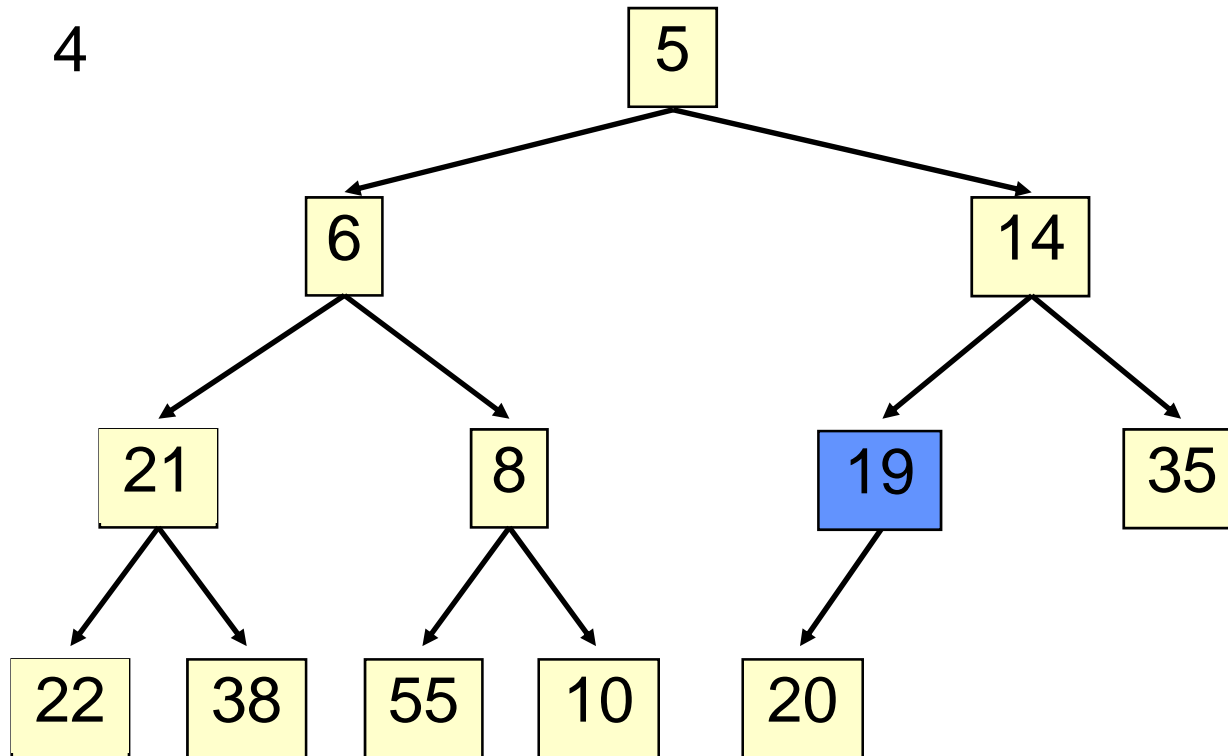
extract()

35



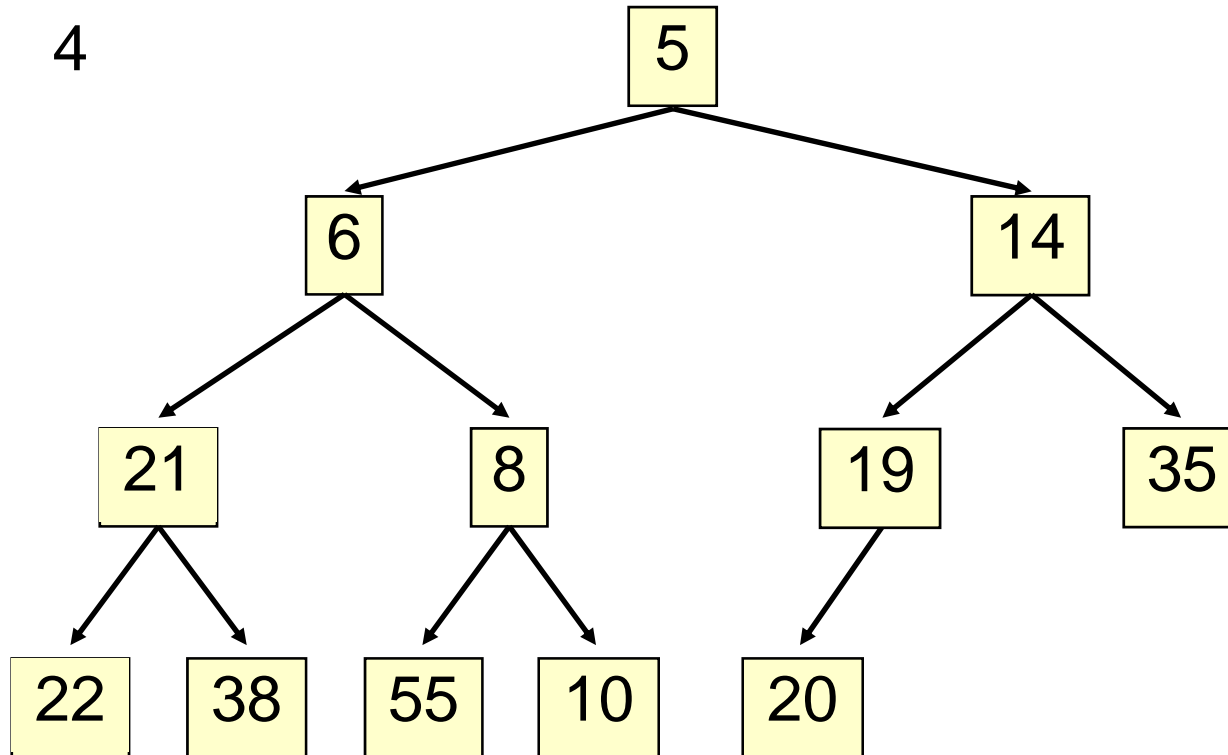
extract()

36



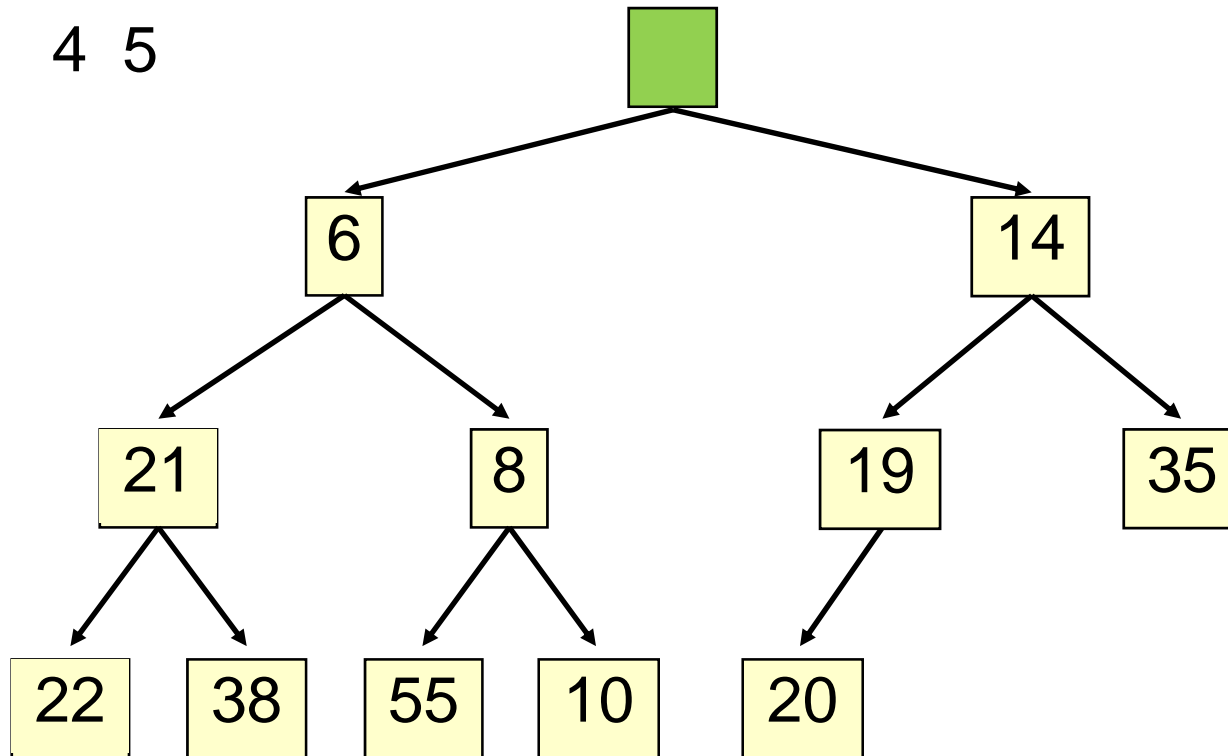
extract ()

37



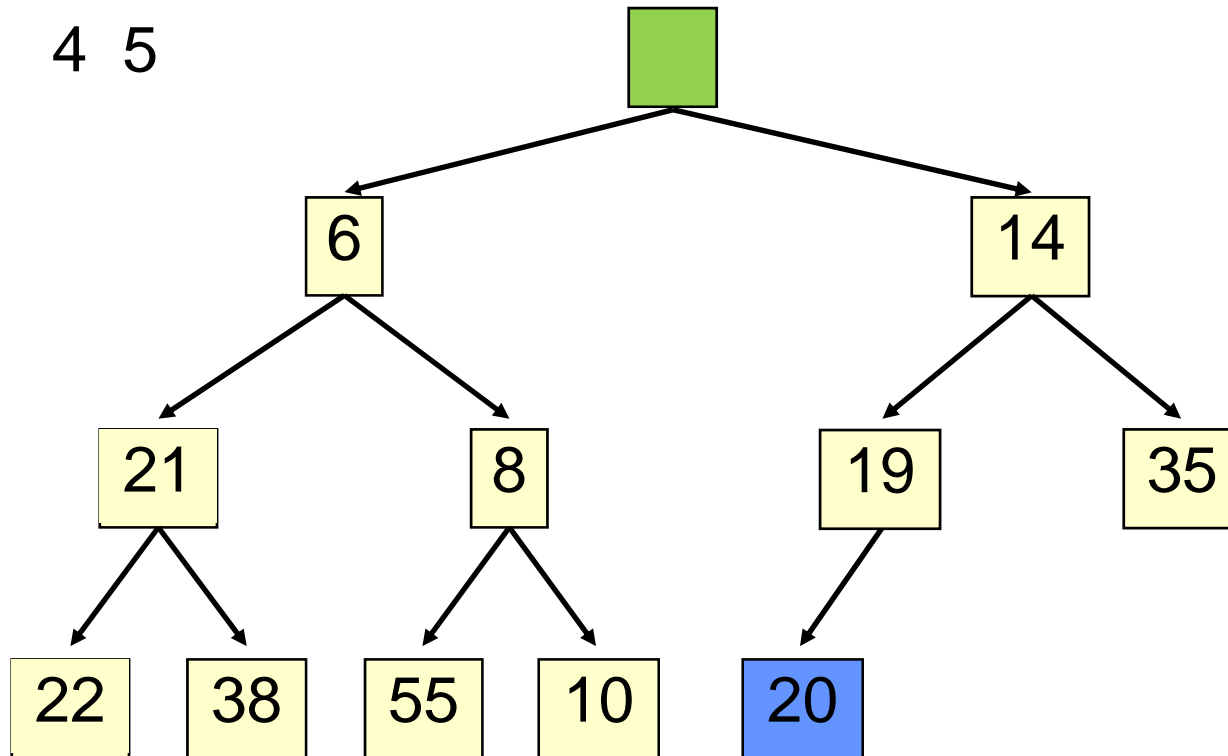
extract()

38



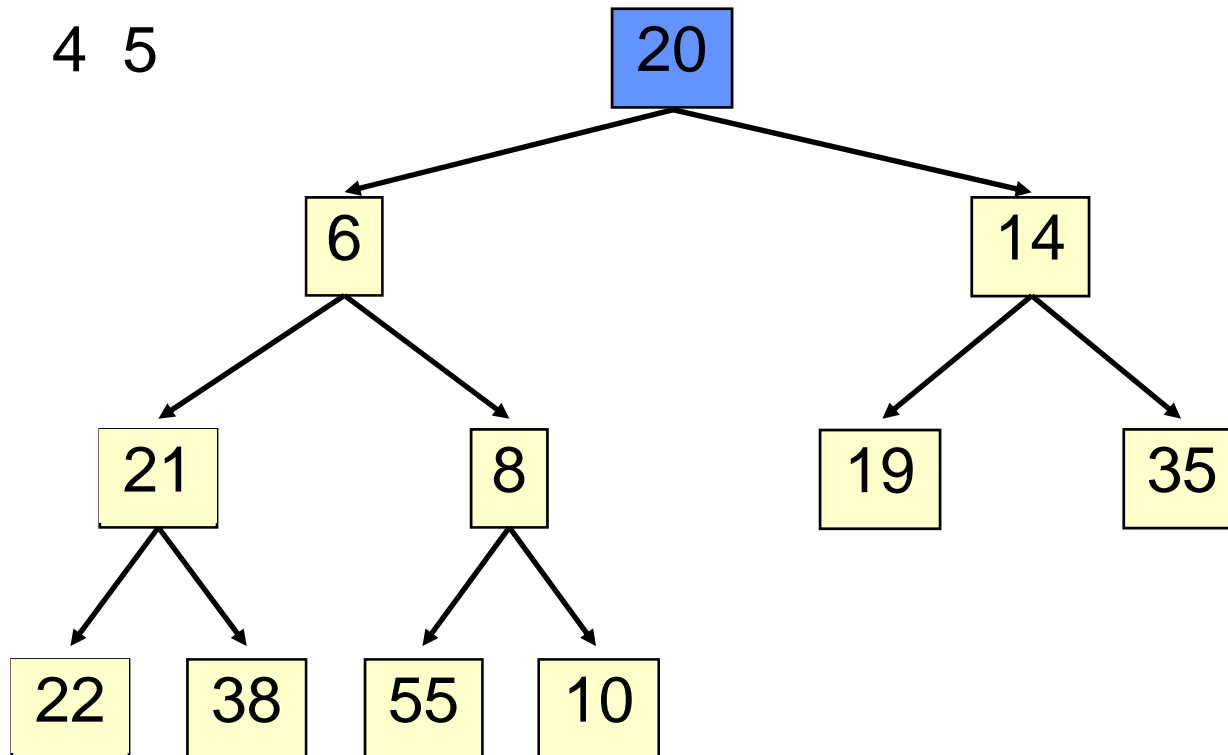
extract()

39



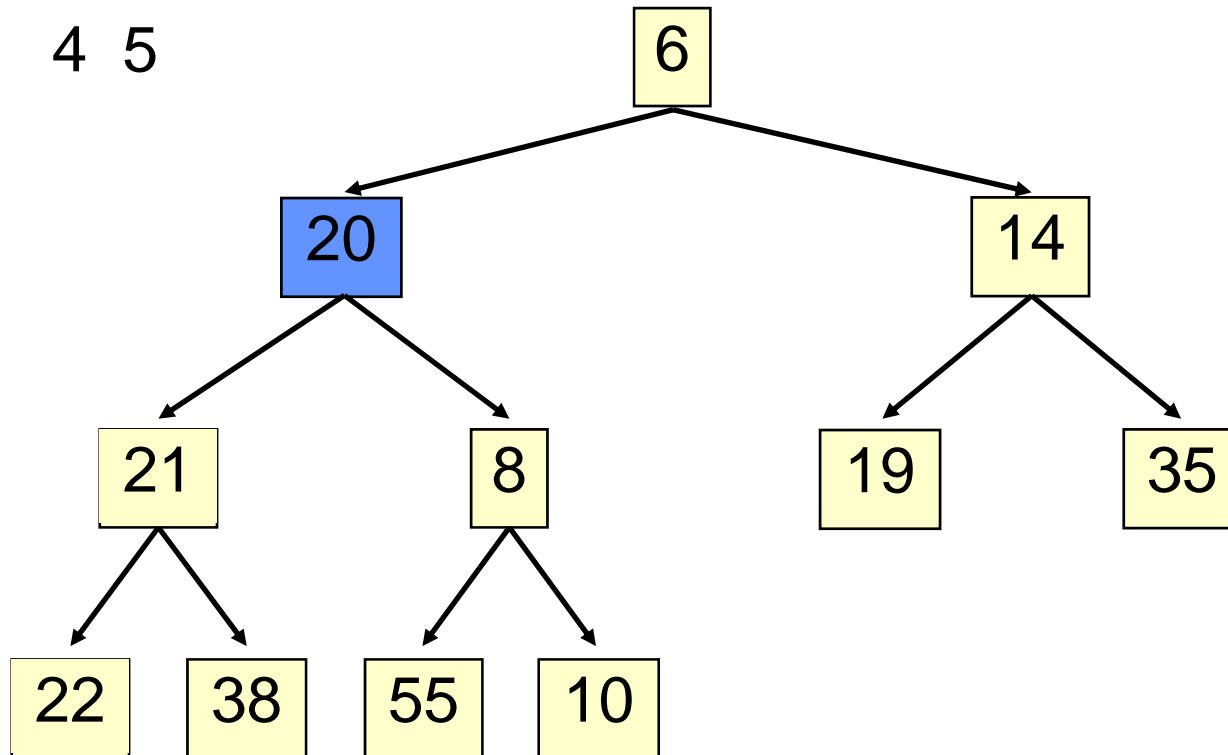
extract ()

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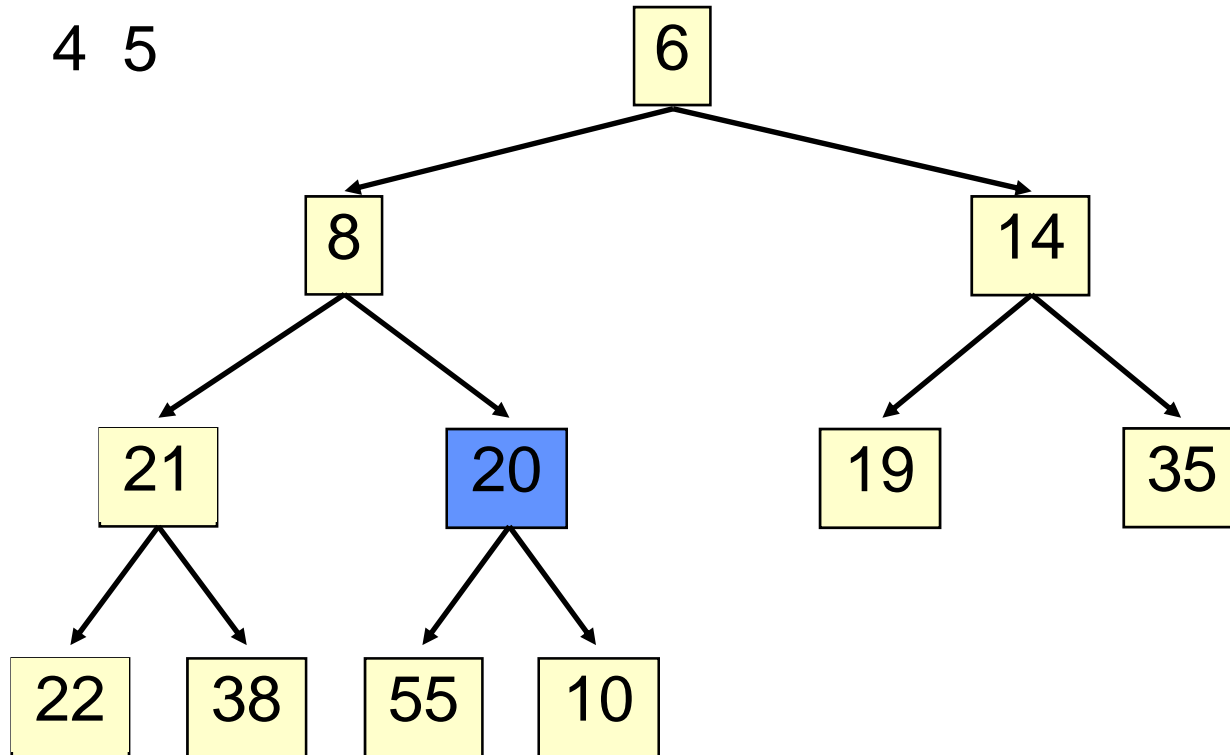
extract ()

41



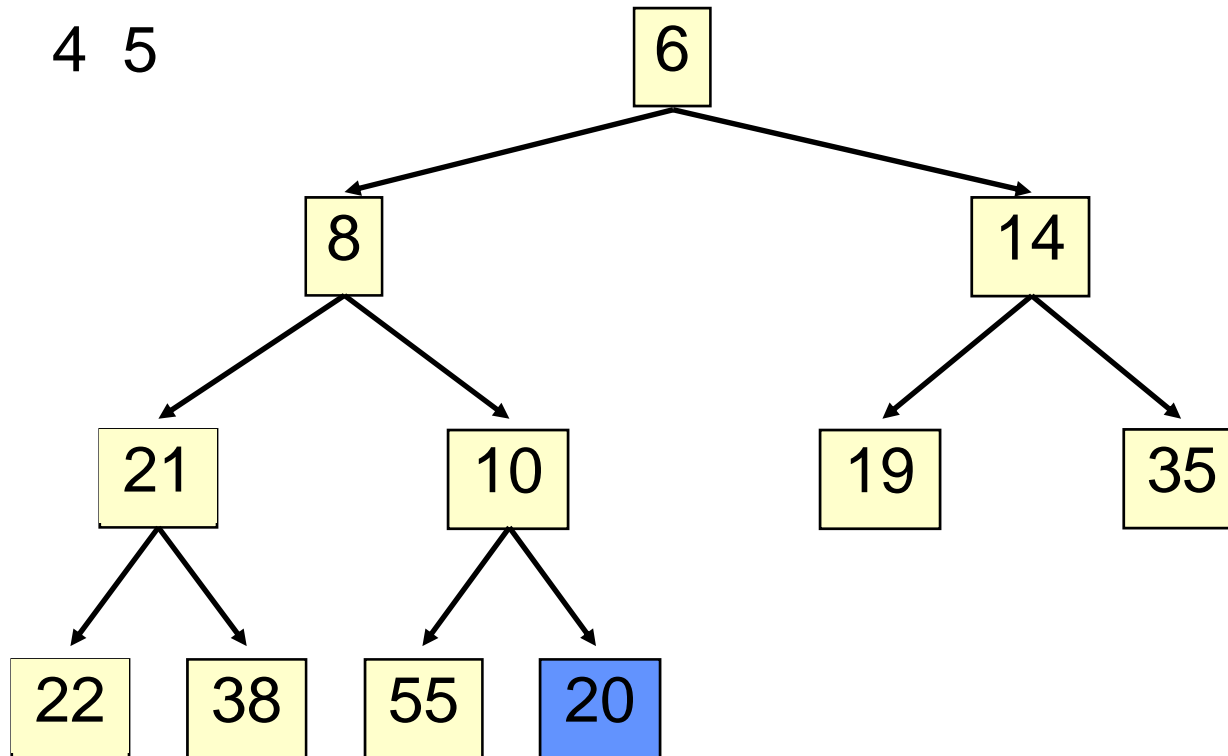
extract()

42



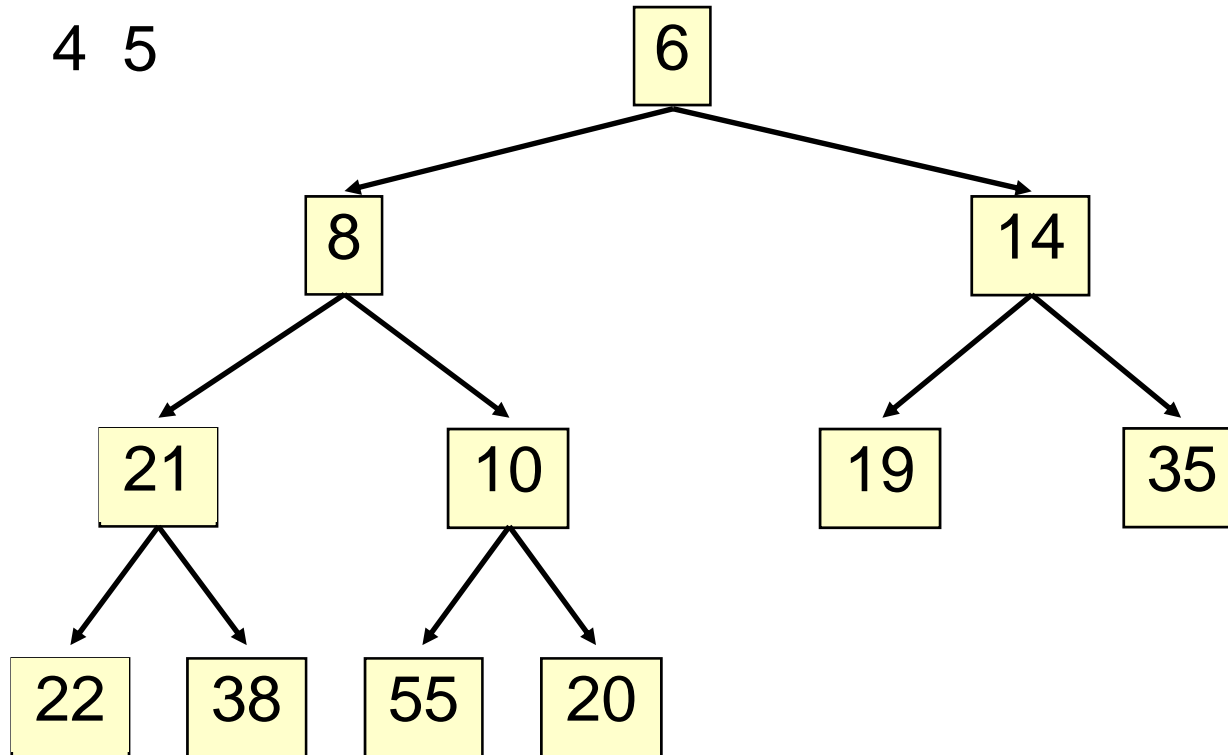
extract()

43



extract ()

44



extract ()

45

- Time is $O(\log n)$, since the tree is balanced

extract()

46

```
public E extract() {
    if (size() == 0) return null;
    E temp = elementAt(0);
    setElementAt(elementAt(size() - 1), 0);
    setSize(size() - 1);
    rotateDown(0);
    return temp;
}
private void rotateDown(int index) {
    int child = 2*(index + 1); //right child
    if (child >= size()
        || elementAt(child - 1).compareTo(elementAt(child)) < 0)
        child -= 1;
    if (child >= size()) return;
    if (elementAt(index).compareTo(elementAt(child)) <= 0)
        return;
    swap(index, child);
    rotateDown(child);
}
```

HeapSort

47

Given a `Comparable[]` array of length n ,

- Put all n elements into a heap – $O(n \log n)$
- Repeatedly get the min – $O(n \log n)$

```
public static void heapSort(Comparable[] a) {  
    PriorityQueue<Comparable> pq  
        = new PriorityQueue<Comparable>();  
    for (Comparable x : a) { pq.insert(x); }  
    for (int i = 0; i < a.length; i++) { a[i] = pq.extract(); }  
}
```

PQ Application: Simulation

48

- Example: Probabilistic model of bank-customer arrival times and transaction times, how many tellers are needed?
 - Assume we have a way to generate random inter-arrival times
 - Assume we have a way to generate transaction times
 - Can simulate the bank to get some idea of how long customers must wait

Time-Driven Simulation

- Check at each *tick* to see if any event occurs

Event-Driven Simulation

- Advance clock to next event, skipping intervening *ticks*
- This uses a PQ!