



Comparisons and the Comparable Interface

Lecture 14
CS211 – Spring 2006

Comparison

- Something that we do a lot
- Can compare all kinds of data with respect to all kinds of comparison relations
 - Identity
 - Equality
 - Order
 - Lots of others

Identity vs. Equality

- For primitive types (e.g., int, long, float, double, boolean)
 - `==` and `!=` are equality tests
- For reference types (i.e., objects)
 - `==` and `!=` are identity tests
 - In other words, they test if the references indicate the *same address* in the Heap
- For equality of objects: use the `equals()` method
 - `equals()` is defined in class `Object`
 - Any class you create inherits `equals` from its parent class, but you can override it (and probably want to)

Identity vs. Equality for Strings

- Quiz: What are the results of the following tests?
 - `"hello".equals("hello")` **true**
 - `"hello" == "hello"` **true**
 - `"hello" == new String("hello")` **false**

Notions of equality

- A is equal to B if A can be substituted for B anywhere
 - Identical things must be equal: `==` implies equals
- Immutable values are equal if they represent same value!
 - `(new Integer(2)).equals(new Integer(2))`
 - `==` is not an abstract operation
- Mutable values can be distinguished by assignment.

```
class Foo { int f; Foo(int g) { f = g; } }
Foo x = new Foo(2);
Foo y = new Foo(2);
```

 - `x.equals(y)`? Not really (`x.f == 1`), but Java fudges equality
- Shallow equality: x equals y if all components are `==`
- Deep equality: x equals y if all components are (deep) equal

Order

- For numeric primitives (e.g., int, float, long, double)
 - Use `<`, `>`, `<=`, `>=`
- For all other reference types
 - `<`, `>`, `<=`, `>=` do not work
 - Not clear you want them to work: suppose we compare `People`
 - ✧ Compare by name?
 - ✧ Compare by height? weight?
 - ✧ Compare by SSN? CUID?
 - Java provides `Comparable` interface
 - Or can use a `Comparator`
- For reference types that correspond to primitive types
 - As of Java 5.0, Java does Autoboxing and Auto-Unboxing of Primitive Types
 - This means, for example, that an `Integer` is automatically converted into an appropriate `int` whenever necessary (and *vice versa*)

Comparable Interface

```
interface Comparable {
    int compareTo(Object x);
}
```

- (Note: this is Java 1.4.2 – Java 5.0 has generics)
- `x.compareTo(y)` returns a negative, zero, or positive integer based on whether `x` is *less-than*, *equal-to*, or *greater-than* `y`, respectively
- *less-than*, *equal-to*, and *greater-than* are *defined* for that class by the implementation of `compareTo`

Example

- To compare people by weight:

```
class Person implements Comparable {
    private int weight;
    ...
    public int compareTo(Object obj) {
        return ((Person)obj).weight - weight;
    }
    public boolean equals(Object obj) {
        return obj instanceof Person &&
            ((Person)obj).weight == weight;
    }
}
```

Consistency

If a class has an `equals` method and also implements `Comparable`, then it is advisable (*but not enforced*) that

```
a.equals(b)
```

exactly when

```
a.compareTo(b) == 0
```

Odd behavior can result if this is violated

Generic Code

- The `Comparable` interface allows generic code for sorting, searching, and other operations that only require comparisons

```
static void mergeSort(Comparable[] a) {...}
static void bubbleSort(Comparable[] a) {...}
```

- The sort methods do not need to know what they are sorting, only how to compare elements

Generic Code Example

- Finding the max element of an array

```
//return max element of an array
static Comparable max(Comparable[] a) {
    //throws ArrayIndexOutOfBoundsException
    Comparable max = a[0];
    for (Comparable x : a) {
        if (x.compareTo(max) > 0) max = x;
    }
    return max;
}
```

- What is the max element? Whatever `compareTo` says it is!

Another Example

- Lexicographic comparison of `Comparable` arrays
- for `int` arrays, `a < b` lexicographically iff either:
 - `a[i] == b[i]` for `i < j` and `a[j] < b[j]`; or
 - `a[i] == b[i]` for all `i < a.length`, and `b` is longer

```
//compare two Comparable arrays lexicographically
static int arrayCompare(Comparable[] a, Comparable[] b)
{
    for (int i = 0; i < a.length && i < b.length; i++) {
        int x = a[i].compareTo(b[i]);
        if (x != 0) return x;
    }
    return b.length - a.length;
}
```

Comparable Interface Update

- Java 5.0 allows the use of “*Generic Types*”

- Aka *parameterized types*
- Here’s the Java 5.0 Comparable interface

```
interface Comparable<T> {  
    int compareTo(T x);  
}
```

- `compareTo` is only defined for arguments of type `T`
 - Attempts to use a different type are caught at *compile time*

Example

- In the Java source code, class `String` looks sort of (other interfaces are also implemented) like this:

```
public final class String  
    implements Comparable<String>{  
    public int compareTo (String s) {...}  
    ...}
```

- Code such as
“`hello`”.`compareTo(new Integer(3))`
generates a compile-time error
 - This implies that the runtime code can be more efficient

Using Comparable for Sorting

- Sorting of arrays provided by Java Collections Framework:

```
import java.util.Arrays;  
...  
String[] names;  
...  
Arrays.sort(names)
```

- This works for arrays of type `comparableType[]` (the base type must implement the Comparable interface)
- (Class `java.util.Arrays` also contains `sort` methods for arrays of type `primitiveType[]` for each primitive type)

Unnatural Sorting

- The ordering given by `compareTo` is considered to be the *natural ordering* for a class
- Sometimes you need to sort based on a different ordering

```
interface Comparator<T> {  
    int compare (T x, T y);  
}
```

- Example: we may normally sort students by CUID, but we might want to produce a list alphabetized by name

- Can use a `Comparator` (a class that implements the `Comparator` interface)
`Arrays.sort(students, comparator)`

- `String`, for example, has a predefined `Comparator`:
`String.CASE_INSENSITIVE_ORDER`

Efficient Programs

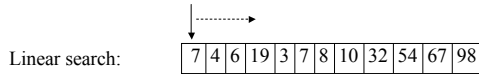
- Have been talking a lot about how to make *writing* programs efficient
 - Interfaces, encapsulation, inheritance, type checking, recursion vs. iteration, ...
- Haven’t talked much about how to make the programs themselves run efficiently
 - How long does it take program to run?
 - Is there an efficient data structure that should be used?
 - Is there a faster algorithm?

Linear Search

- Input:
 - Unsorted array `A` of Comparables
 - Value `v` of type Comparable
- Output:
 - True if `v` is in array `A`, false otherwise
- Algorithm: examine the elements of `A` in some order until you either
 - Find `v`: return true, or
 - You have unsuccessfully examined all the elements of the array: return false

Code for Linear Search

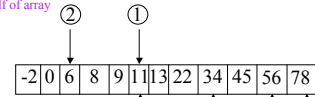
```
// Linear search on possibly unsorted array
public static boolean linearSearch(Comparable[] a, Object v) {
    for (int i = 0; i < a.length; i++)
        if (a[i].compareTo(v) == 0) return true;
    return false;
}
```



Binary Search

- **Input:**
 - Sorted array $A[0..n-1]$ of Comparable
 - Value v of type Comparable
- **Output:**
 - True if v is in array A , false otherwise
- **Algorithm:** similar to looking up telephone directory
 - Let m be the middle element of the array
 - If ($m = v$) return true
 - If ($m < v$) search right half of array
 - If ($m > v$) search left half of array

Search for 6



Search for 94



```
// lo and hi are the two end points of interval of array
public static boolean binarySearch(Comparable[] a, int lo, int hi, Object v) {
    int middle = (lo + hi)/2;
    int c = A[middle].compareTo(v);

    // Base cases
    if (c == 0) return true;
    // Check if array interval has only one element
    if (lo == hi) return false;

    // Array interval has more than one element, so continue searching
    if (c > 0) return binarySearch(a, lo, middle - 1, v); // Left half
    else return binarySearch(a, middle + 1, hi, v); // Right half
}
```

Invocation: assume array named data contains values

..... `binarySearch(data, 0, data.length - 1, v)`.....

Comparing Algorithms

- If you run binary search and linear search on a computer, you will find that binary search runs much faster than linear search
- Stating this precisely can be quite subtle
- One approach: asymptotic complexity of programs
 - Big-O analysis
- Two steps:
 - Compute running time of program
 - Running time \Rightarrow asymptotic running time

Running Time of an Algorithm

- In general, running time of a program such as linear search depends on many factors
 - Machine on which program is executed
 - Laptop vs. supercomputer
 - Size of input (array A)
 - Big array vs. small array
 - Values in array and value we search for
 - v is first element examined in array vs. v is not in array
- To talk precisely about running times of programs, we must specify all three factors above

Defining an Algorithm's Running Time

1. Machine on which algorithm (i.e., program) is executed
 - Random-access Memory (RAM) model of computing
 - Measure of running time: number of operations executed
 - Other models used in CS: Turing machine, Parallel RAM model, ...
 - Simplified RAM model for now:
 - Each data comparison is one operation.
 - All other operations are free.
 - Evaluate searching/sorting algorithms by estimating number of comparisons they execute
 - ❖ It can be shown that, for comparison-based searching and sorting algorithms, the total number of operations executed on RAM model is proportional to number of data comparisons executed

Defining Running Time (cont'd)

2. Dependence on size of input

- Rather than compute a single number, we will compute a function from problem size to number of comparisons
 - E.g., $f(n) = 32n^2 - 2n + 23$ where n is problem size
- Each program has its own measure of problem size
- For searching/sorting, natural measure is size of array on which you are searching/sorting

Defining Running Time (cont'd)

3. Dependence of running time on input values

- Consider set I_n of all possible inputs of size n
- Find number of comparisons for each possible input in this set
- Compute
 - Average: usually hard to compute
 - Worst-case: easier to compute
- We will use worst-case complexity

$([3,6], 2)$ $([-4,5], -9)$
 $([3,6], 3)$

Possible inputs of size 2 for linear/binary search

Computing Running Times

Linear search:

7	4	6	19	3	7	8	10	32	54	67	98
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Assume array is of size n .
 Worst-case number of comparisons: v is not in array.
 Number of comparisons = n .
 Running time of linear search: $T_L(n) = n$

Binary search: sorted array of size n

-2	0	6	8	9	11	13	22	34	45	56	78
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Worst-case number of comparisons: v is not in array.

$$T_B(n) = \lfloor \log_2(n) \rfloor + 1$$