Recursion
(by Nicholas Ruozzi and Lisa Minich)

Recursion is similar to induction in reverse. We want to take the kth state and reduce it to the $(k-1)^{th}$ state. For example:

Summing up the numbers 0 through k can be done iteratively:

```
int sum =0;
for(int i=1; i< =k; i++)
{
  sum +=  i;
}
```

or recursively:

```
public int sum(int n)
{
   if(n==0)      //base case
     return 0;
   else            //recursive case
     return sum(n-1) + n;
}
```

Where the nth state is added to the $(n-1)^{th}$ state for all n greater than the base case. Which in this example was 0.

Tail Recursion:
A method is tail recursive if the last action of the recursive method is the recursive call. For example, the solutions to problems 1 and 2 are not tail recursive, but the solution to problem 3 is tail recursive. Because recursion builds frame upon frame of the stack, the additional overhead required by non-tail recursive functions could be costly for large inputs.

Problem 1 (Reverse a string recursively):

```
  public String reverseString(String word)
  {
    if(word == null || word.equals(""))
      return word;
    else
      return reverseString(word.substring(1, word.length())) + word.substring(0,1);
  }
```

Problem 2 (Remove consecutive duplicates from a string recursively):

For example, convert "aabccba" to "abcba"

```
  public String removeDuplicates(String word)
  {
    if(word == null || word.length() <= 1)
      return word;
    else if( word.charAt(0) == word.charAt(1) )
      return removeDuplicates(word.substring(1, word.length()));
    else
      return word.charAt(0) + removeDuplicates(word.substring(1, word.length()));
  }
```

Problem 3 (Compute n mod m without using %):

```
    public int modulus(int val, int divisor)
    {
      if(val < divisor)
        return val;
      else
        return modulus(val - divisor, divisor);
    }
```

Problem 1 (again) with Tail Recursion:

Someone asked in section if it was possible to convert non-tail recursive functions into tail recursive ones. The answer is usually yes. To illustrate how this can be done, I will rewrite problem one using tail recursion. To do this, I will utilize a helper function.

```
  public String reverseString(String word)
  {
      return tailReverse(word, "");
  }

  public String tailReverse(String word, String res)
  {
    if(word == null || word.equals(""))
      return res;
    else
      return tailReverse(word.substring(1, word.length()), res + word.charAt(0));
  }
```

tailReverse is a tail recursive method that reverses a string.

Parsing:
All of the information on parsing can be found in the lecture notes.