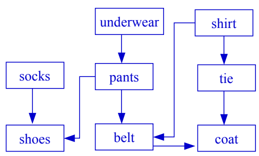
Special Graphs

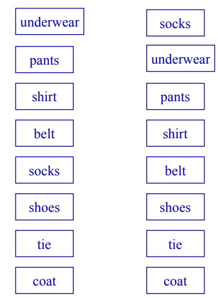
- Some graph types are used so often that they have special names
  - undirected graph
  - directed graph
  - tree
  - dag (*directed acyclic graph*)
    - acyclic ≡ no cycles
    - cycle ≡ path that starts and ends at the same vertex (i.e., a loop)
- Example dag:



- Suppose you have a directed graph; how do you tell if it's a dag?

Topological Sort

- Claim: A directed graph is a dag iff it can be topologically sorted
- A *topological sort* of a directed graph is an ordering of the vertices such that if there is an edge from u to v then u appears before v in the ordering
- There are usually multiple solutions for topological sort
- Example solutions:



Algorithm for Topological Sort

count[v] = number of edges *into* vertex v
W = set of all vertices
while W nonempty:
    Choose u in W with count[u] = 0
    Remove u from W
    for v such that (u,v) is an edge:
        count[v] = count[v] – 1

- If the graph is not acyclic then the "Choose" step will fail
- Correctness
  - We can find a vertex that is acceptable as the *first* vertex
  - Once we remove that vertex (and its edges) we have a new topological sort problem of smaller size

Runtime for Topological Sort

count[v] = number of edges *into* vertex v
W = set of all vertices
while W nonempty:
    Choose u in W with count[u] = 0
    Remove u from W
    for v such that (u,v) is an edge:
        count[v] = count[v] – 1

- If the graph is not acyclic then the "Choose" step will fail
- Adjacency Matrix
  - while loop is executed |V| times
  - Choose step: O(|V|)
  - Decrementing count: O(|V|)
  - Total time: $O(|V|^2)$
- Adjacency List
  - while loop is executed |V| times
  - Choose step: O(1)
    - Use a stack of waiting 0-count vertices
  - Decrementing count: O(|E|) over all
  - Total time: O(|E| + |V|)