Image transformations, Part 2

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Administrivia

- Assignment 4 has been posted
  - Due the Friday after spring break
- TA evaluations
  - http://www.engineering.cornell.edu/TAEval/survey.cfm
- Midterm course evaluations

Tricks with convex hull

- What else can we do with convex hull?
- Answer: sort!

- Given a list of numbers (x_1, x_2, ... x_n), create a list of 2D points:

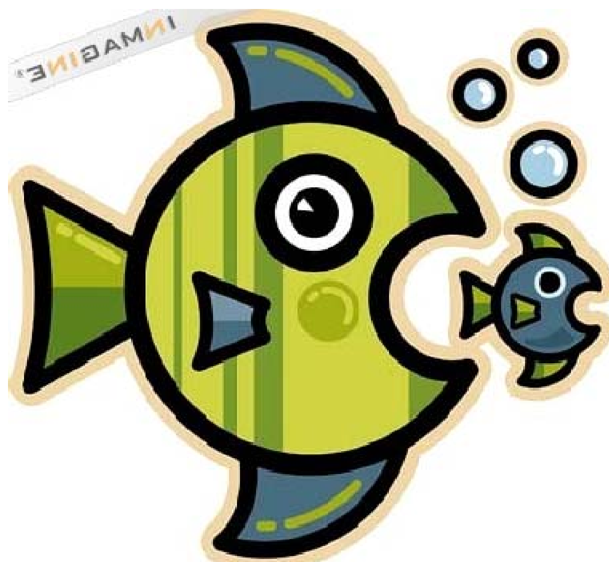
  (x_1, x_1^2), (x_2, x_2^2), ... (x_n, x_n^2)

- Find the convex hull of these points – the points will be in sorted order
- What does this tell us about the running time of convex hull?

Tricks with convex hull

- This is called a *reduction* from sorting to convex hull

- We saw a reduction once before



Last time: image transformations

2D Linear Transformations

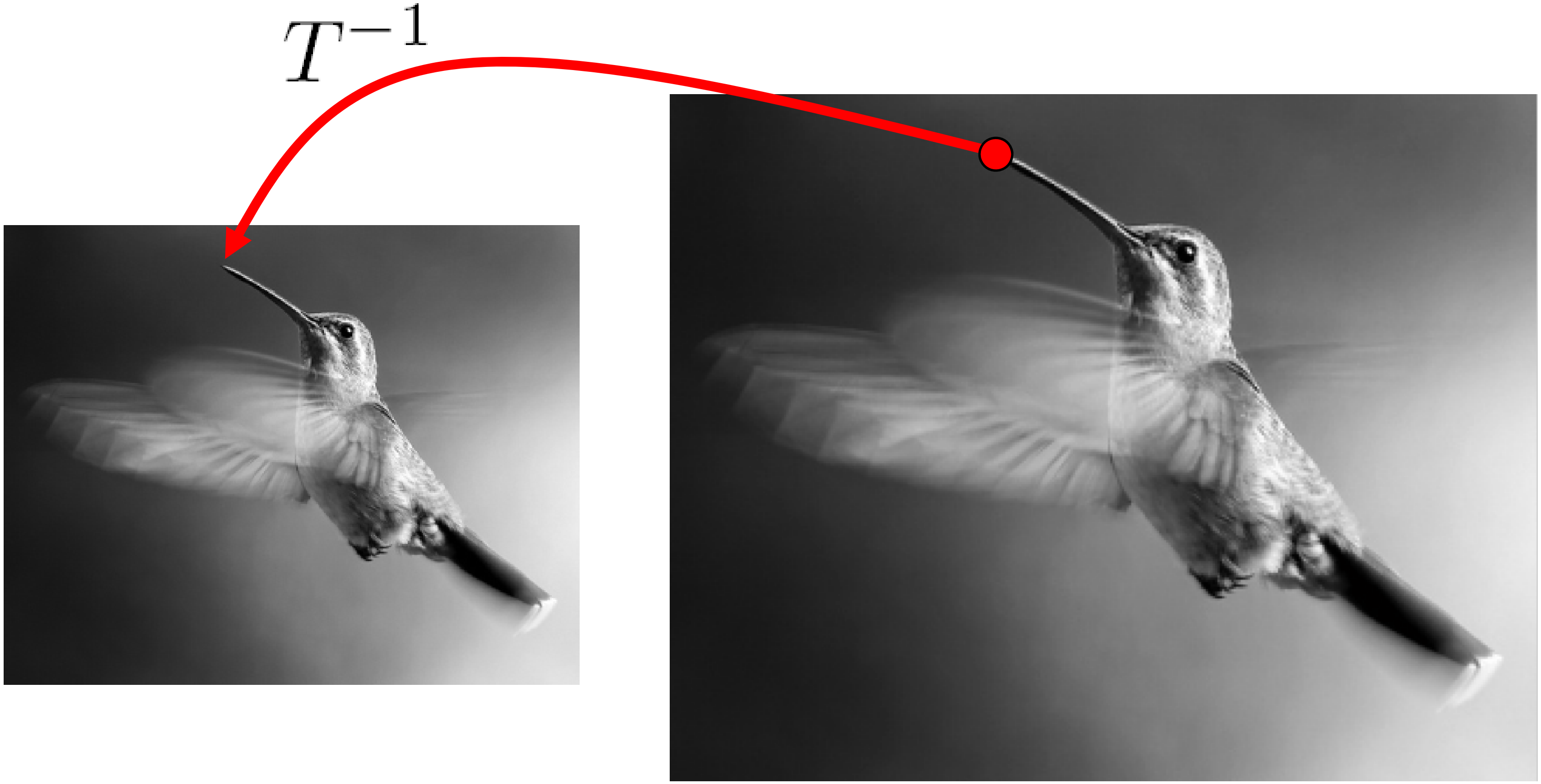
- Can be represented with a 2D matrix

$$T = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

- And applied to a point using matrix multiplication

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} ax + by \\ cx + dy \end{bmatrix}$$

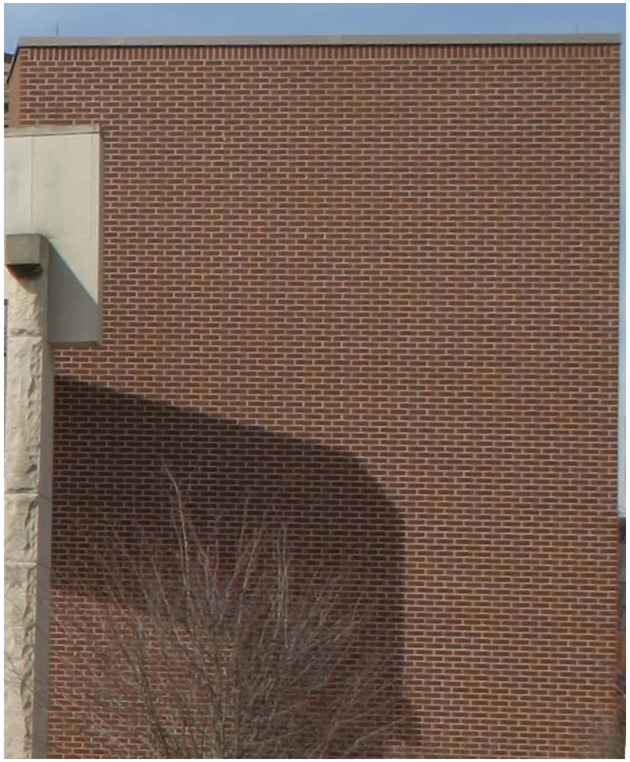
Inverse mapping



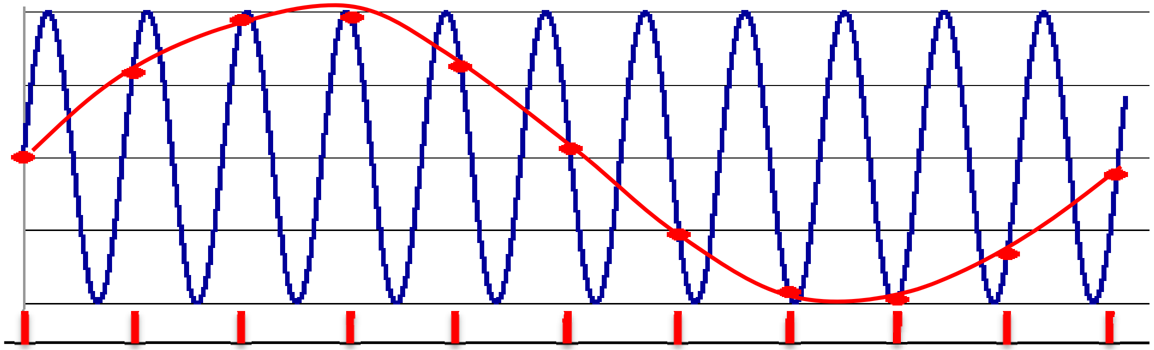
Downsampling

- Suppose we scale image by 0.25

Downsampling



What's going on?



- **Aliasing** can arise when you sample a continuous signal or image
- Occurs when the sampling rate is not high enough to capture the detail in the image
- Can give you the wrong signal/image—an alias

Examples of aliasing

- Wagon wheel effect

- Moiré patterns

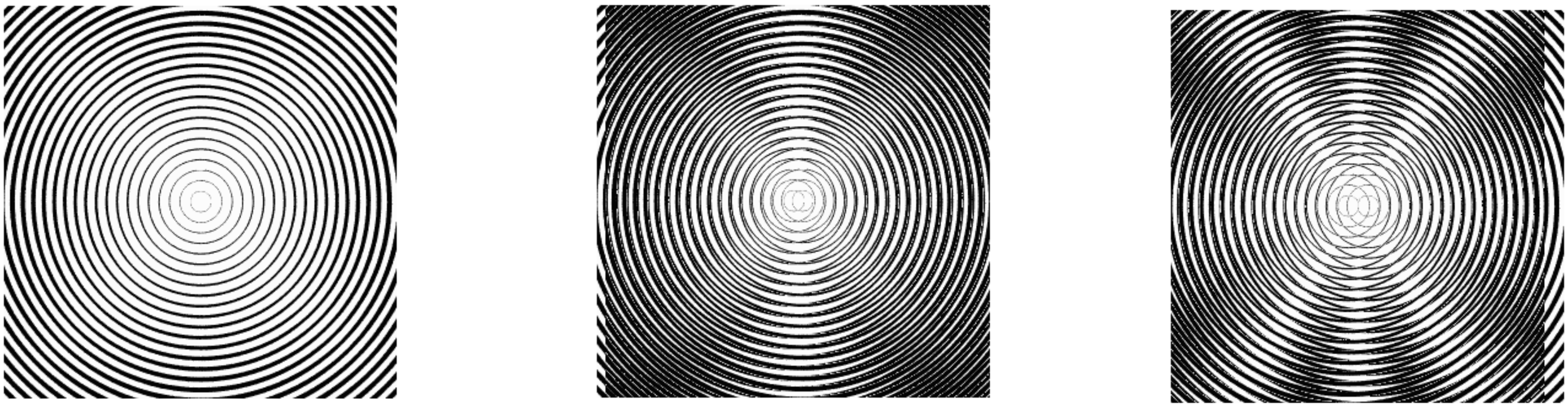
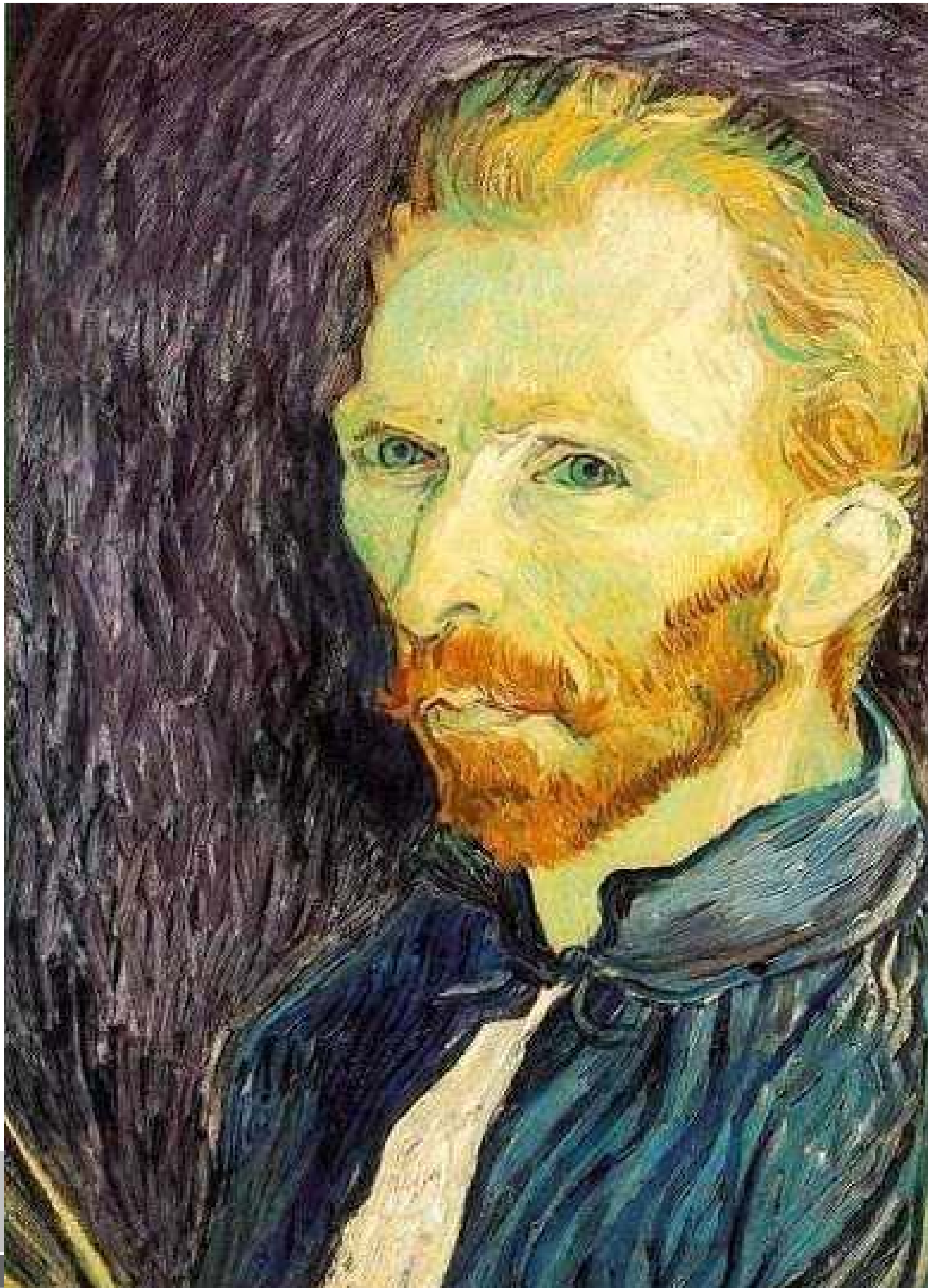


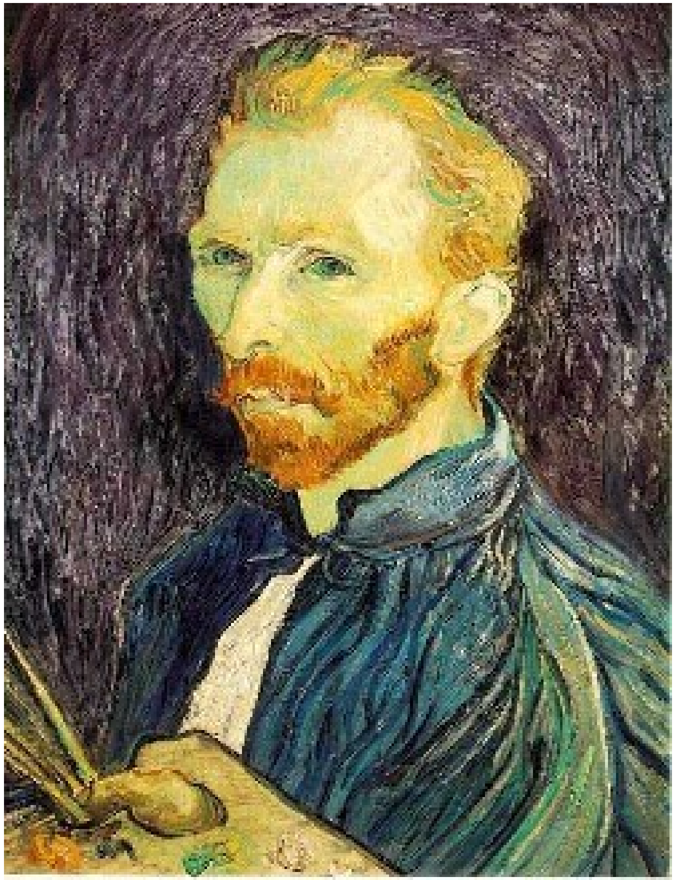
Image credit: Steve Seitz

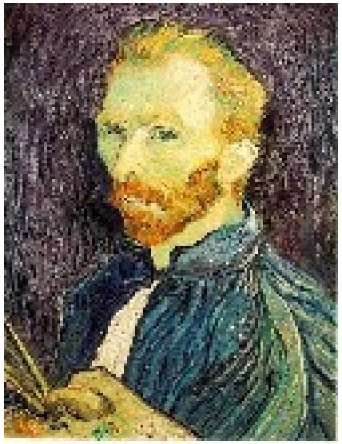
- This image is too big to fit on the screen. How can we create a half-sized version?



Slide credits: Steve Seitz

Image sub-sampling



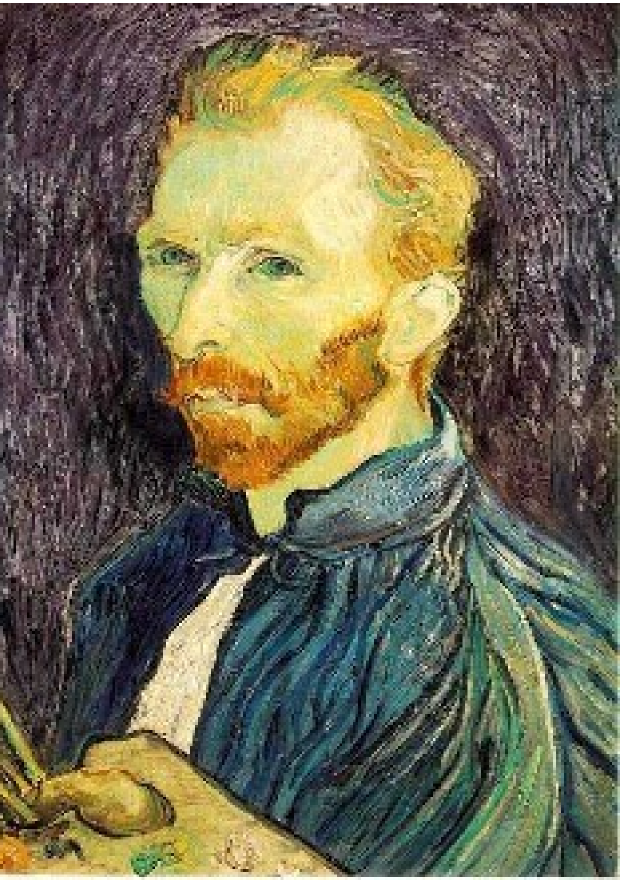


1/4

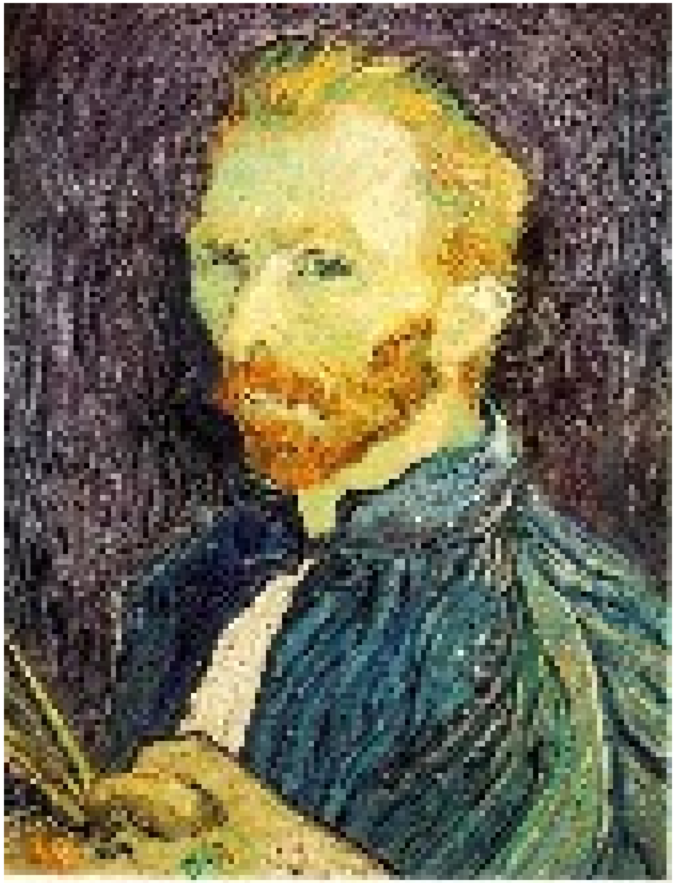
1/8

Current approach: throw away every other row and column (subsample)

Image sub-sampling



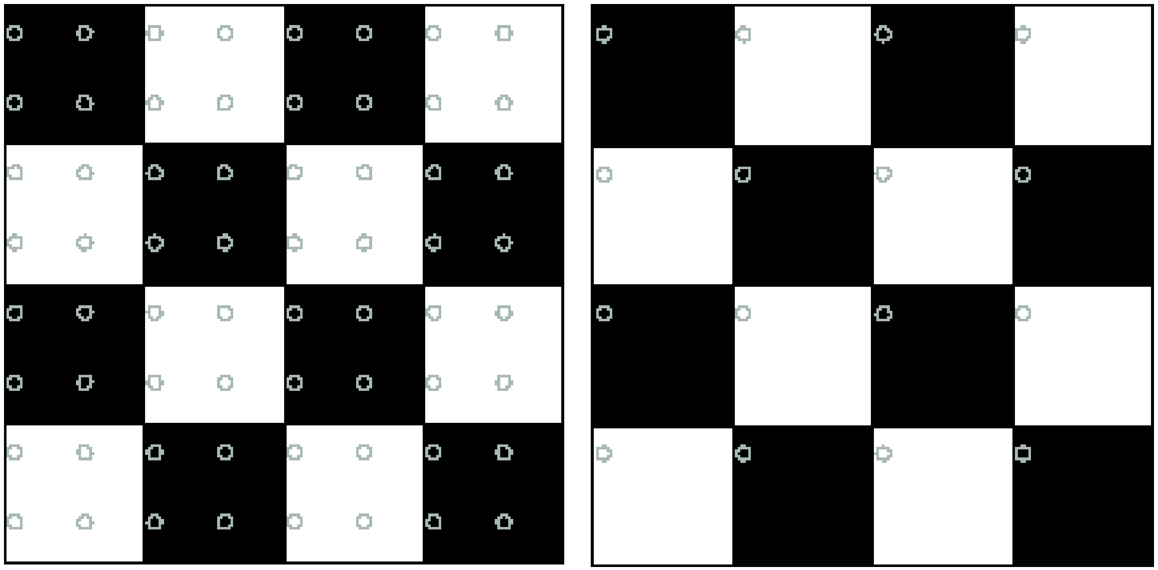
•*1/2*



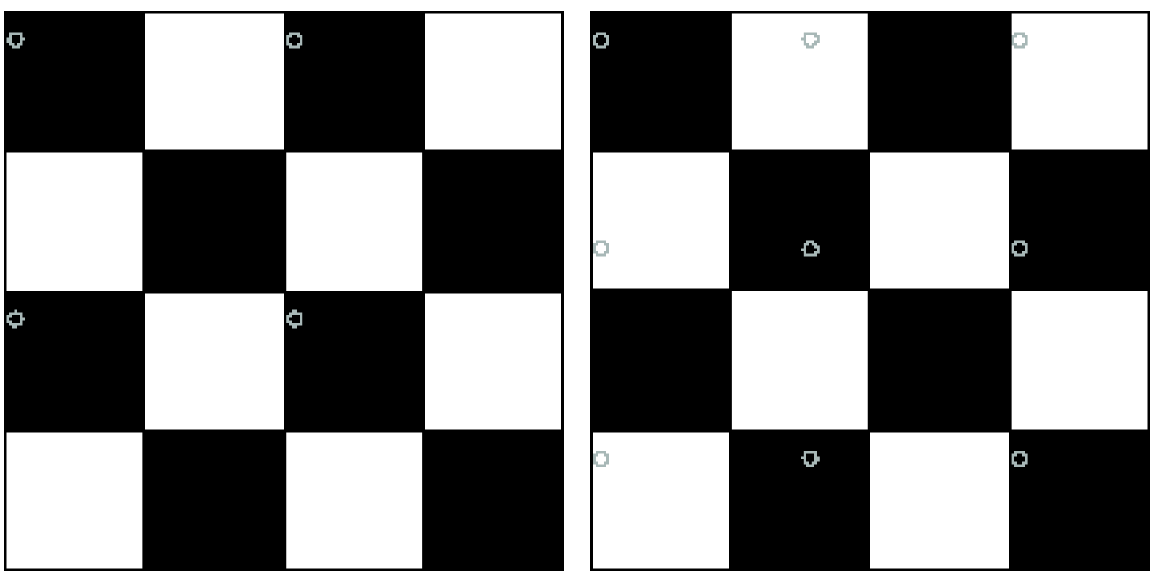
•*1/4* *(2x zoom)*

•*1/8* *(4x zoom)*

2D example



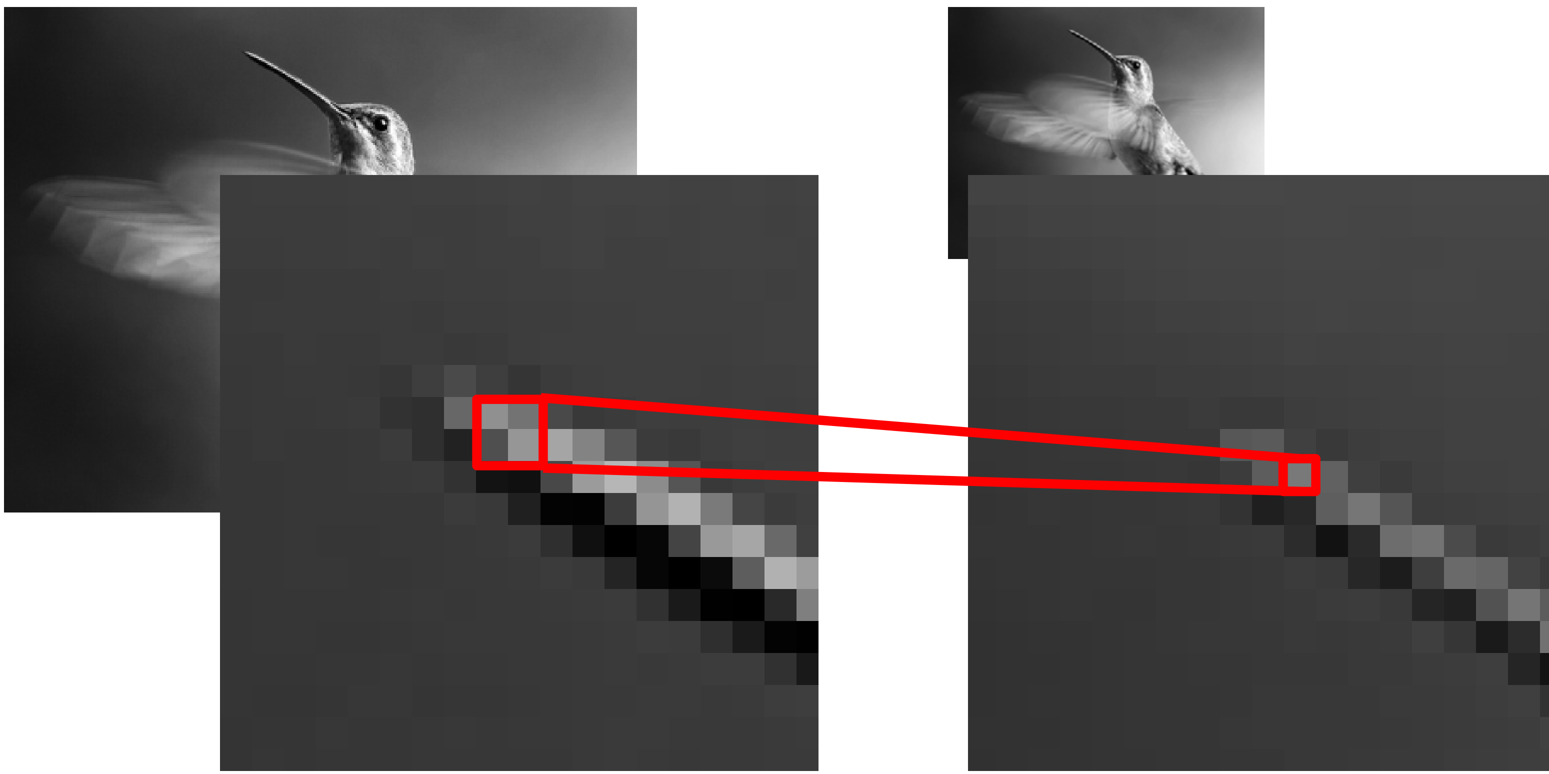
Good sampling



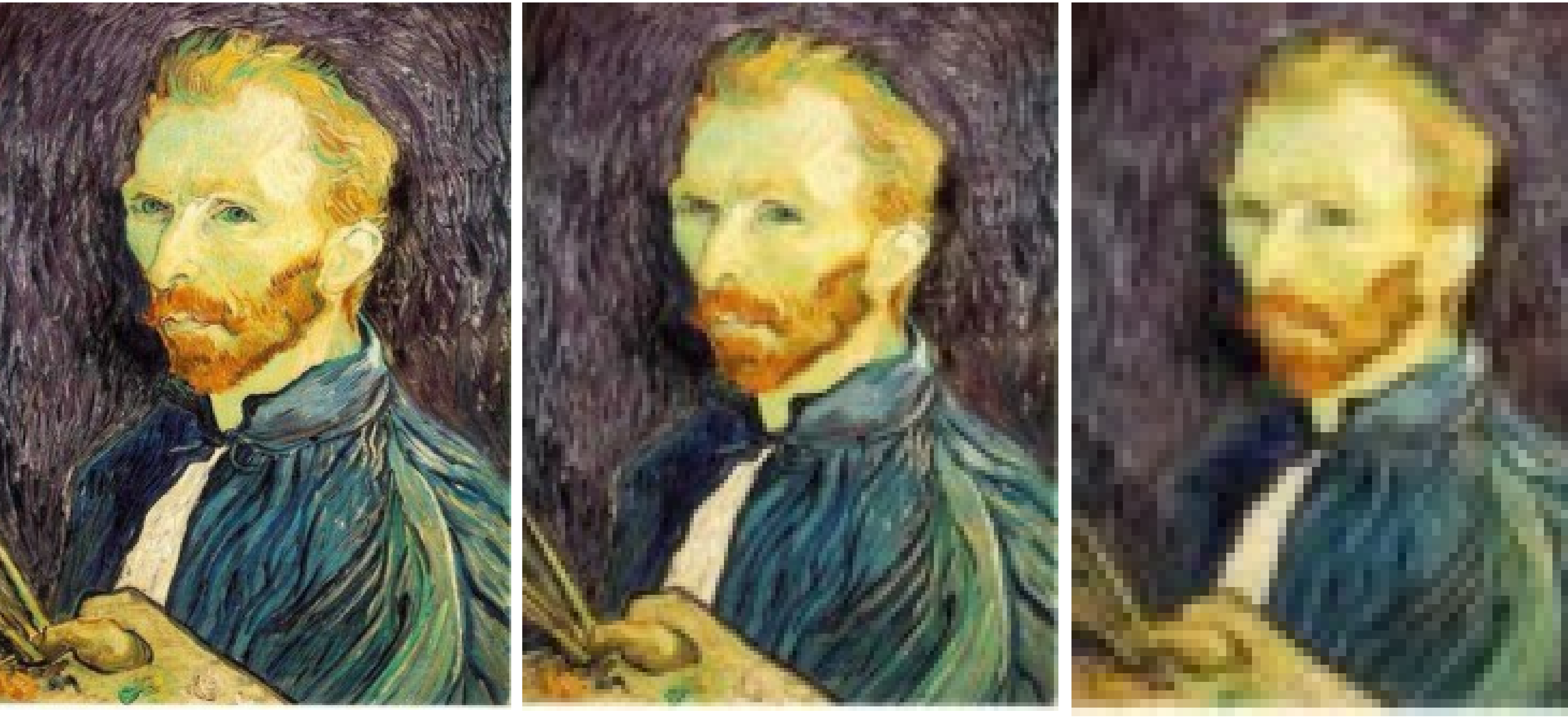
Bad sampling

Image sub-sampling

- What's really going on?



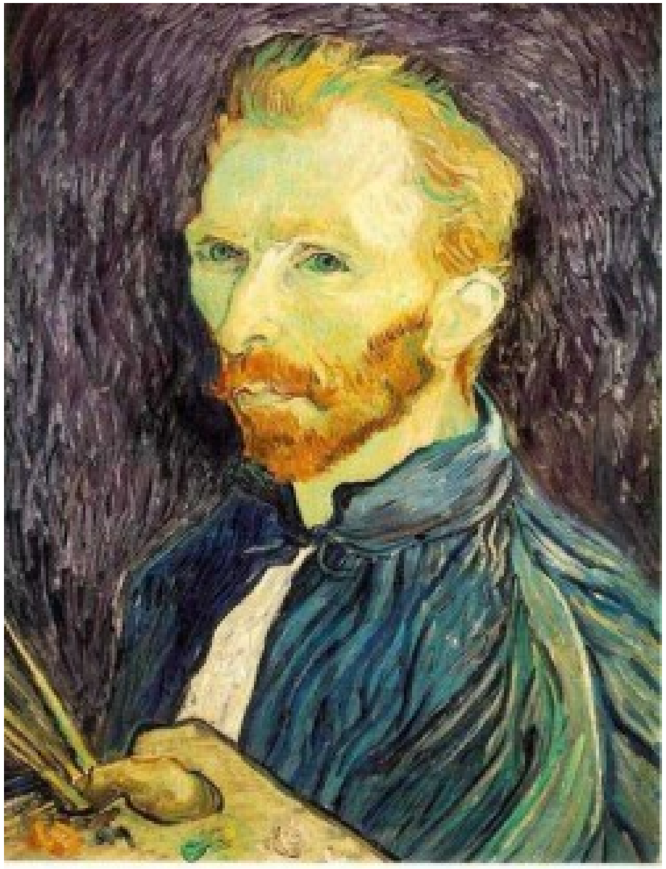
Subsampling with pre-filtering



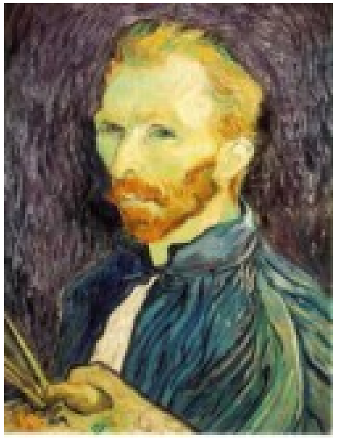
Average 2x2 *Average 4x4* *Average 8x8*

- Solution: blur the image, then subsample
  - Filter size should double for each ½ size reduction.

Subsampling with pre-filtering



Average 2x2

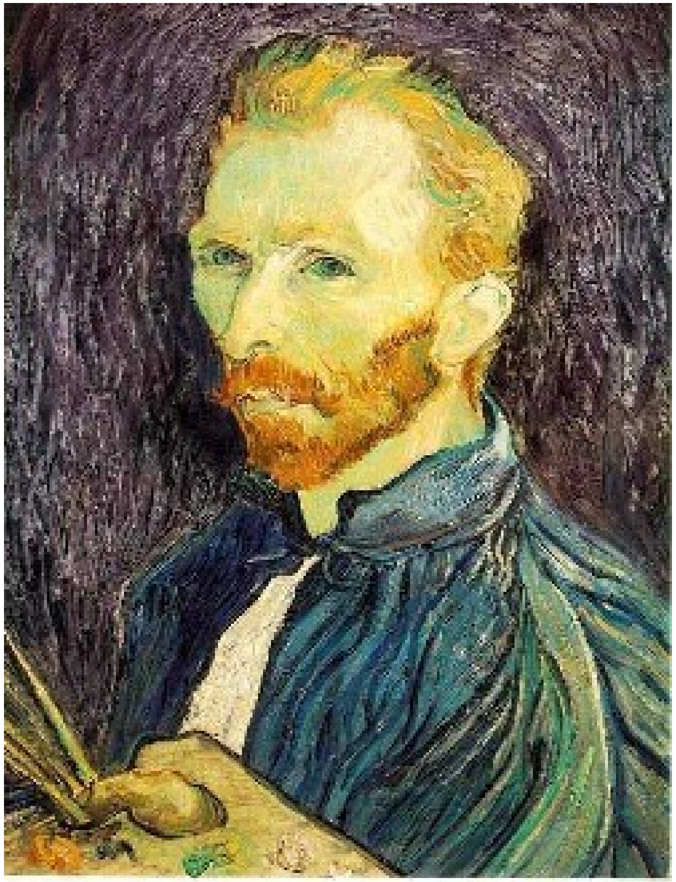


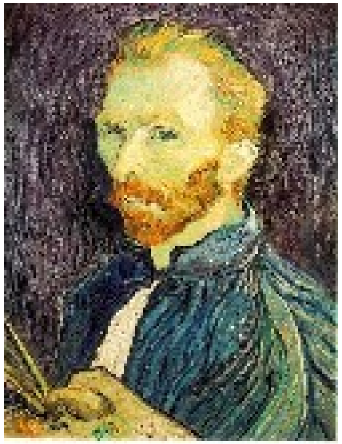
Average 4x4

Average 8x8

- Solution: blur the image, then subsample
  - Filter size should double for each ½ size reduction.

Compare with





1/4

1/8

Recap: convolution

- "Filtering"
- Take one image, the *kernel* (usually small), slide it over another image (usually big)
- At each point, multiply the kernel times the image, and add up the results
- This is the new value of the image

0	1/5	0
1/5	1/5	1/5
0	1/5	0

Blurring using convolution

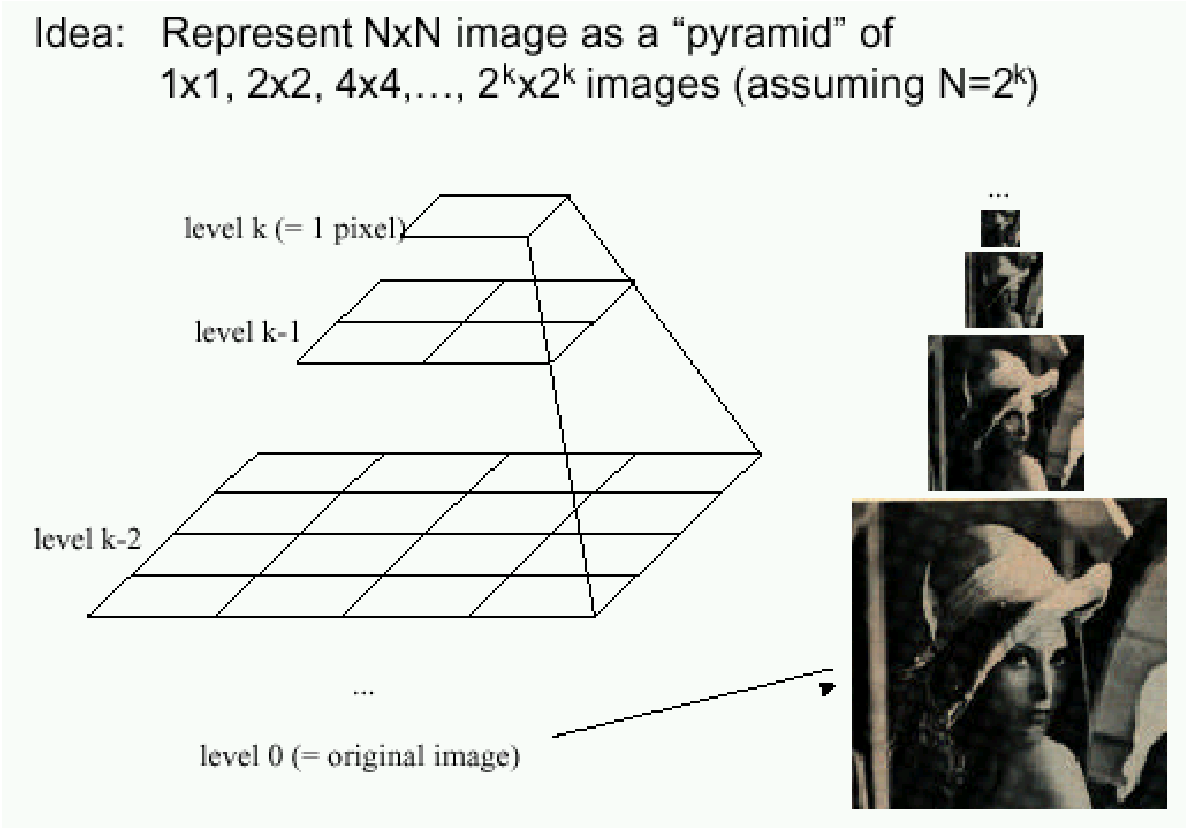
- 2x2 average kernel

1/4	1/4
1/4	1/4

- 4x4 average kernel

1/8	1/8	1/8	1/8
1/8	1/8	1/8	1/8
1/8	1/8	1/8	1/8
1/8	1/8	1/8	1/8

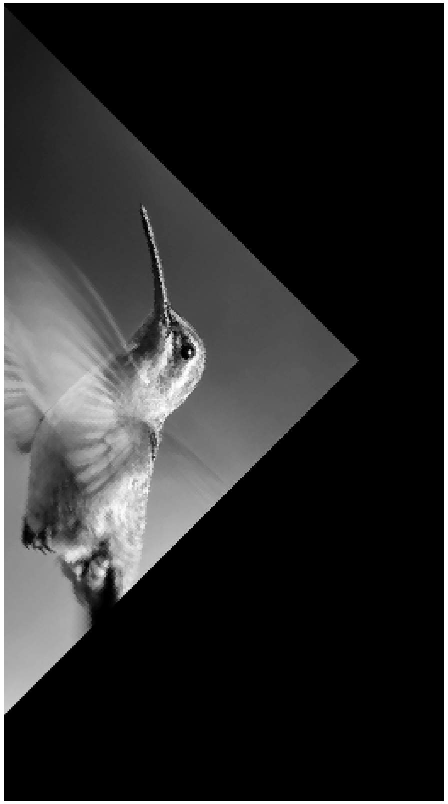
Sometimes we want many resolutions



- Known as a **Gaussian Pyramid** [Burt and Adelson, 1983]
  - In computer graphics, a mip map [Williams, 1983]
  - A precursor to wavelet transform

Back to image transformations

- Rotation is around the point (0, 0) – the upper-left corner of the image



- This isn't really what we want…

Translation

- We really want to rotate around the *center* of the image

- Approach: move the center of the image to the origin, rotate, then the center back

- (Moving an image is called "translation")

- But translation isn't linear...

Homogeneous coordinates

- Add a 1 to the end of our 2D points

  $(x, y) \rightarrow (x, y, 1)$

- "Homogeneous" 2D points

- We can represent transformations on 2D homogeneous coordinates as 3D matrices

Translation

$$T = \begin{bmatrix} 1 & 0 & s \\ 0 & 1 & t \\ 0 & 0 & 1 \end{bmatrix}$$

- Other transformations just add an extra row and column with [0 0 1]

$$S = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad R = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

scale *translation*

Correct rotation

- Translate center to origin

$$T_1 = \begin{bmatrix} 1 & 0 & -w/2 \\ 0 & 1 & -h/2 \\ 0 & 0 & 1 \end{bmatrix}$$

- Rotate

$$R = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_2 R T_1$$

- Translate back to center

$$T_2 = \begin{bmatrix} 1 & 0 & w/2 \\ 0 & 1 & h/2 \\ 0 & 0 & 1 \end{bmatrix}$$