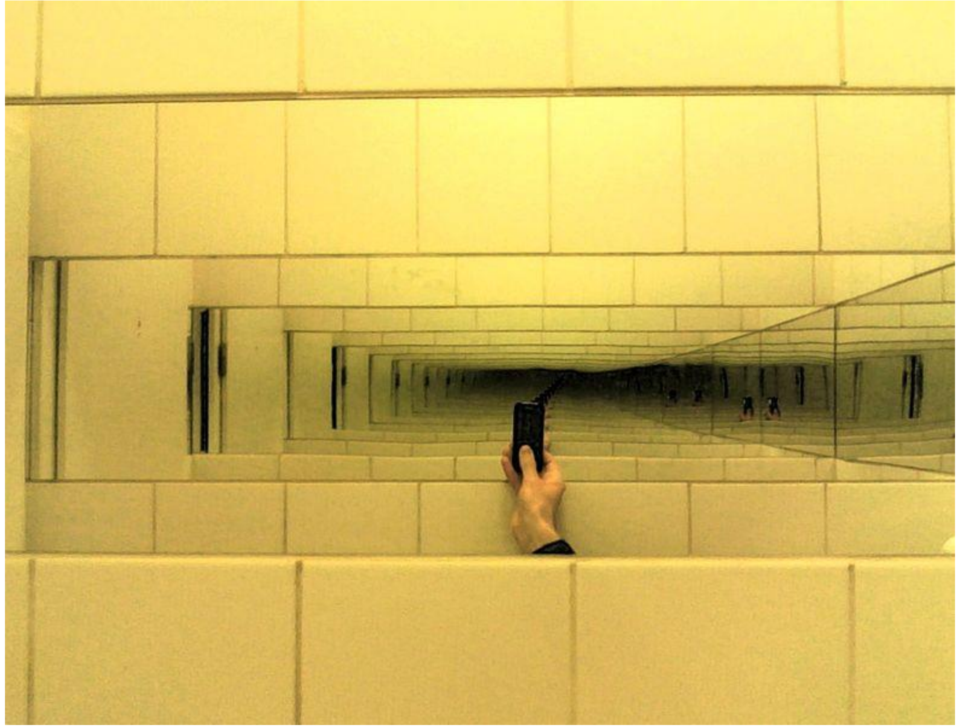
- Previous Lecture:
  - OOP: Access modifiers & inheritance
- Today, Lecture 25:
  - Recursion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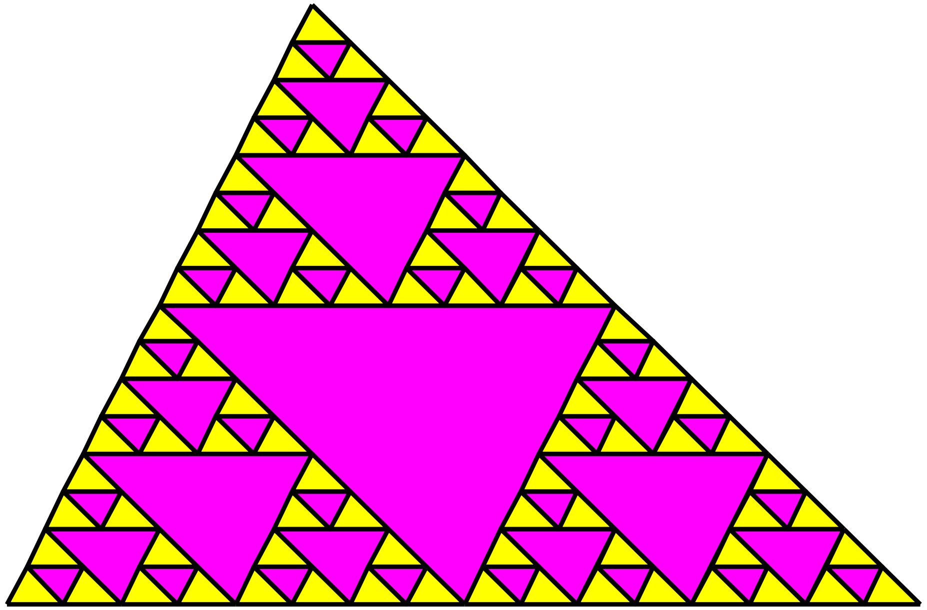
- Announcements:
  - Project 6A code is available
    - Description still being refined, but draft is available
    - More reading than writing
  - Final exam May 24
    - Conflict survey coming this weekend; reply ASAP
  - Course evaluation survey next week
    - Anonymous responses, but credit for submitting it

Recursion

A method of problem solving by breaking a problem into smaller and smaller instances of the <u>same</u> problem until an instance is so small that it’s trivial to solve



Fibonacci sequence

Sequence

$$f_1 = 1, \qquad f_2 = 1$$
$$f_n = f_{n-1} + f_{n-2}$$

```
f(1)= 1; f(2)= 1
for k = 3:n
    f(k)= f(k-1) + f(k-2);
end
```

Function

$$f(n) = \begin{cases} 1, & n < 3 \\ f(n-1) + f(n-2), & n \geq 3 \end{cases}$$

```
function y = f(n)
    if n < 3
        y = 1;
    else
        y = f(n-1) + f(n-2);
    end
end
```

Recursion

- The Fibonacci sequence is defined recursively:

  F(1)=1, F(2)=1,
  F(3)= F(1) + F(2) = 2
  F(4)= F(2) + F(3) = 3

  F(k) = F(k-2) + F(k-1)

  It is defined in terms of itself; its definition invokes itself.

- Algorithms, and functions, can be recursive as well. I.e., a function can call itself.

- Example: remove all occurrences of a character from a string

  `'gc aatc gga c '` → `'gcaatcggac'`

Example: removing all occurrences of a character

- Can solve using iteration—check one character (one component of the vector) at a time

	1	2	…	k	…		
s	`'c'`	`'s'`	`' '`	`'1'`	`'1'`	`'1'`	`'2'`

Subproblem 1:
Keep or discard s(1)

Subproblem 2:
Keep or discard s(2)

Subproblem k:
Keep or discard s(k)

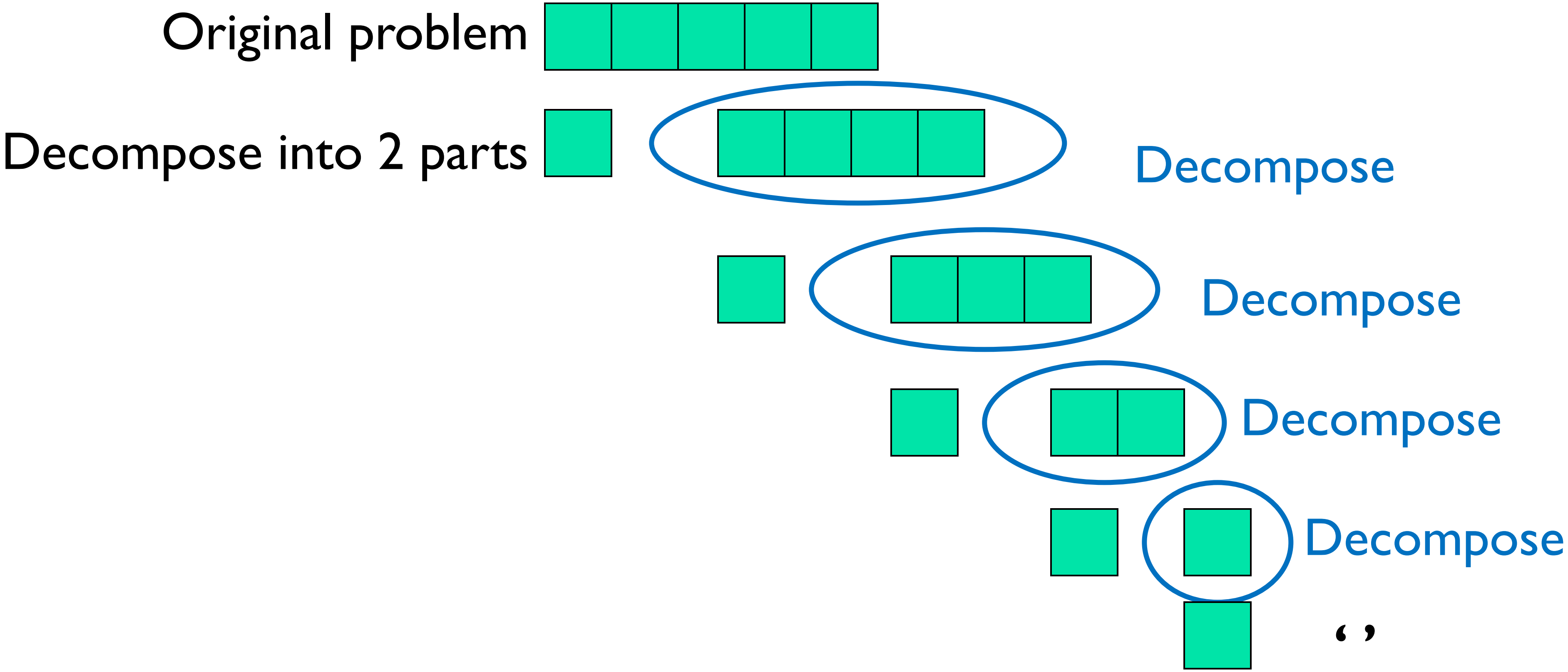
Iteration:
Divide problem into sequence of equal-sized, identical subproblems

See `RemoveChar_loop.m`

Example: removing all occurrences of a character

- Can solve using ***recursion***
  - Original problem: remove all the blanks in string s
  - Decompose into two parts: *1*. remove blank in s(1)

    2. remove blanks in s(2:length(s))



```
function s = removeChar(c, s)
% Return string s with character c removed

if length(s)==0  % Base case: nothing to do
    return
else









end
```

```
function s = removeChar(c, s)
% Return string s with character c removed

if length(s)==0  % Base case: nothing to do
    return
else
  if s(1)~=c
    % return string is
    % s(1) and remaining s with char c removed

  else
    % return string is just
    % the remaining s with char c removed

  end
end
```

```
function s = removeChar(c, s)
% Return string s with character c removed

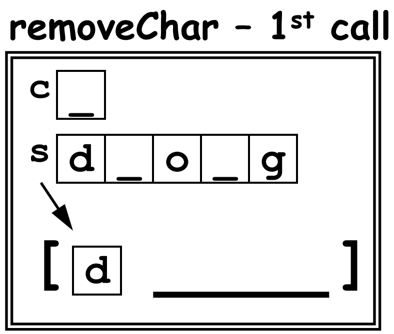
if length(s)==0  % Base case: nothing to do
    return
else
  if s(1)~=c
    % return string is
    % s(1) and remaining s with char c removed
    s= [s(1)                              ];
  else
    % return string is just
    % the remaining s with char c removed
    s=                              ;
  end
end
```

```
function s = removeChar(c, s)
% Return string s with character c removed

if length(s)==0  % Base case: nothing to do
    return
else
  if s(1)~=c
    % return string is
    % s(1) and remaining s with char c removed
    s= [s(1) removeChar(c, s(2:length(s)))];
  else
    % return string is just
    % the remaining s with char c removed
    s= removeChar(c, s(2:length(s)));
  end
end
```

```
function s = removeChar(c, s)
if length(s)==0
  return
else
  if s(1)~=c
    s= [s(1) removeChar(c, s(2:length(s)))];
  else
    s= removeChar(c, s(2:length(s)));
  end
end
```

removeChar('_', 'd_o_g')



```
function s = removeChar(c, s)
if length(s)==0
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else
  if s(1)~=c
3 1 s= [s(1) removeChar(c, s(2:length(s)))];
  else
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  end
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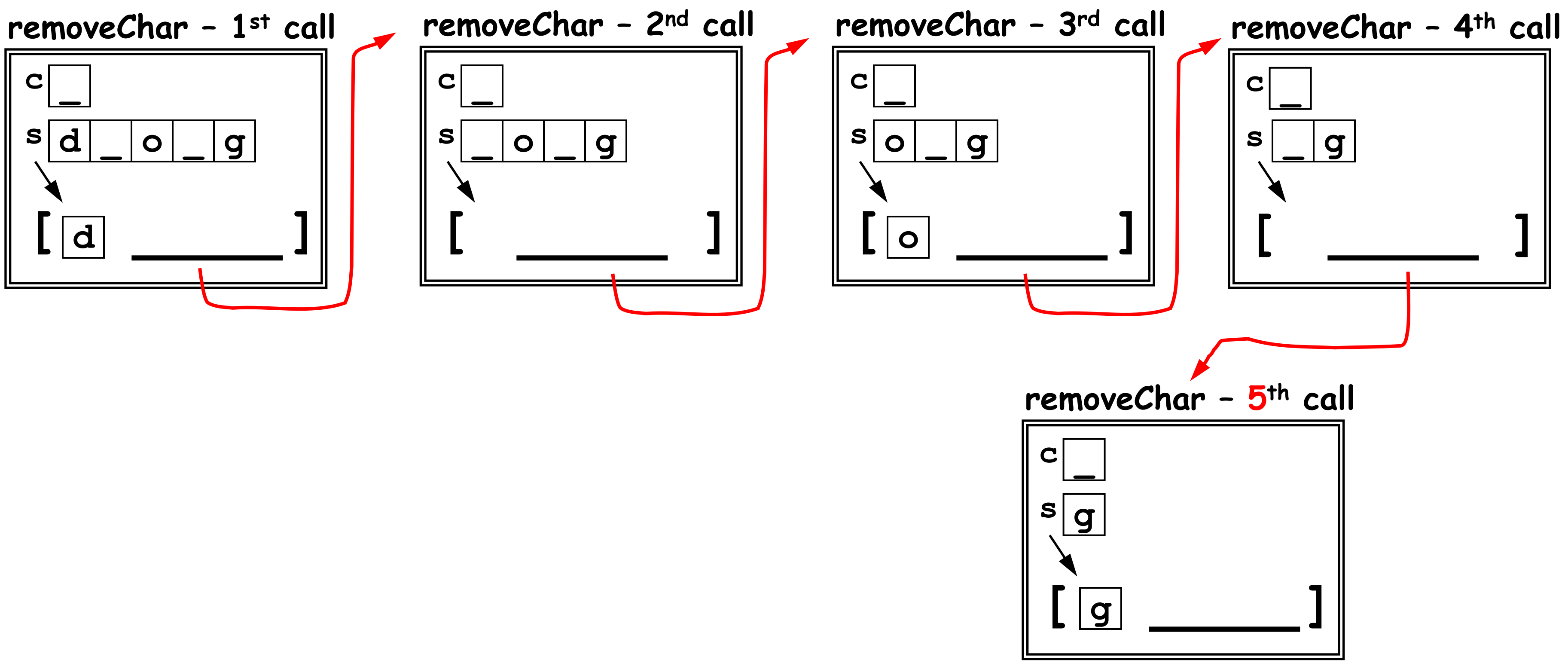
```

removeChar('_', 'd_o_g')



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5 3 1 s= [s(1) removeChar(c, s(2:length(s)))];
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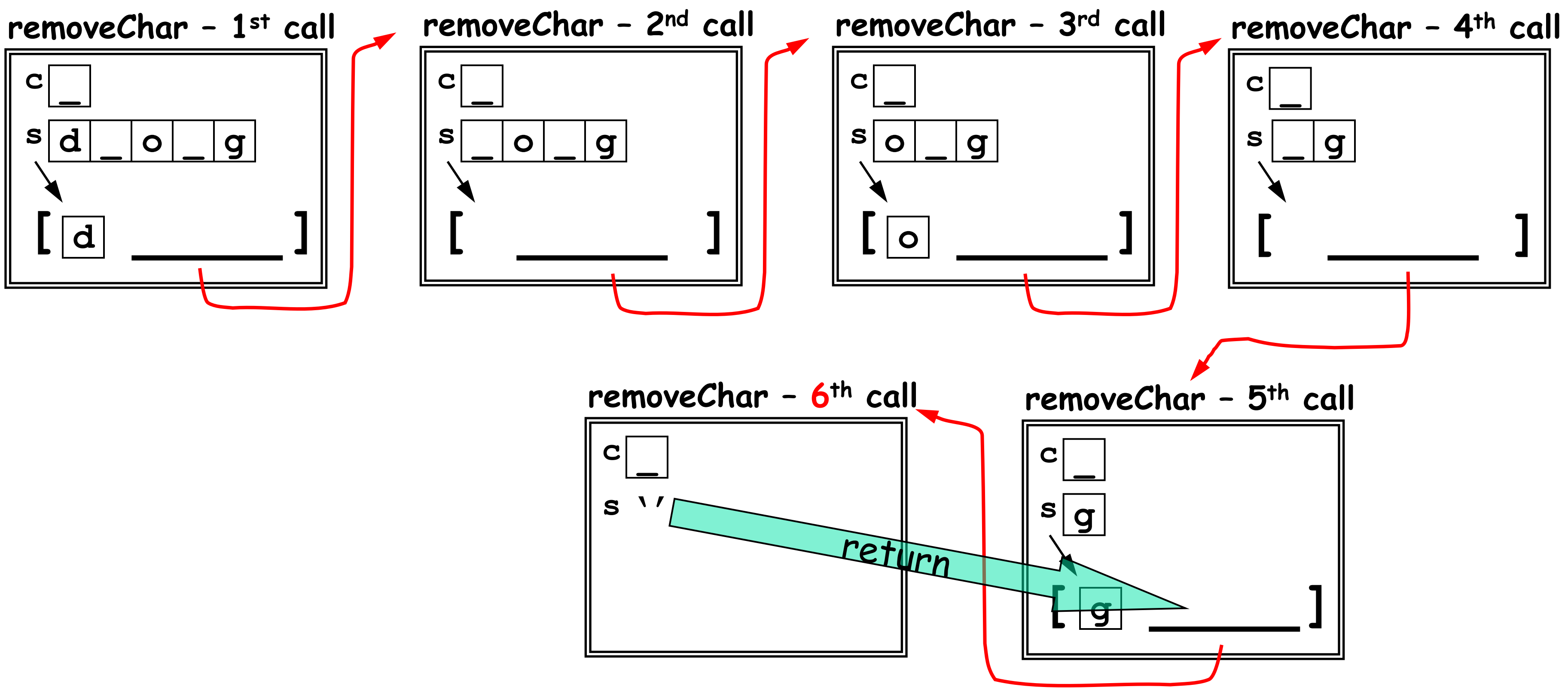

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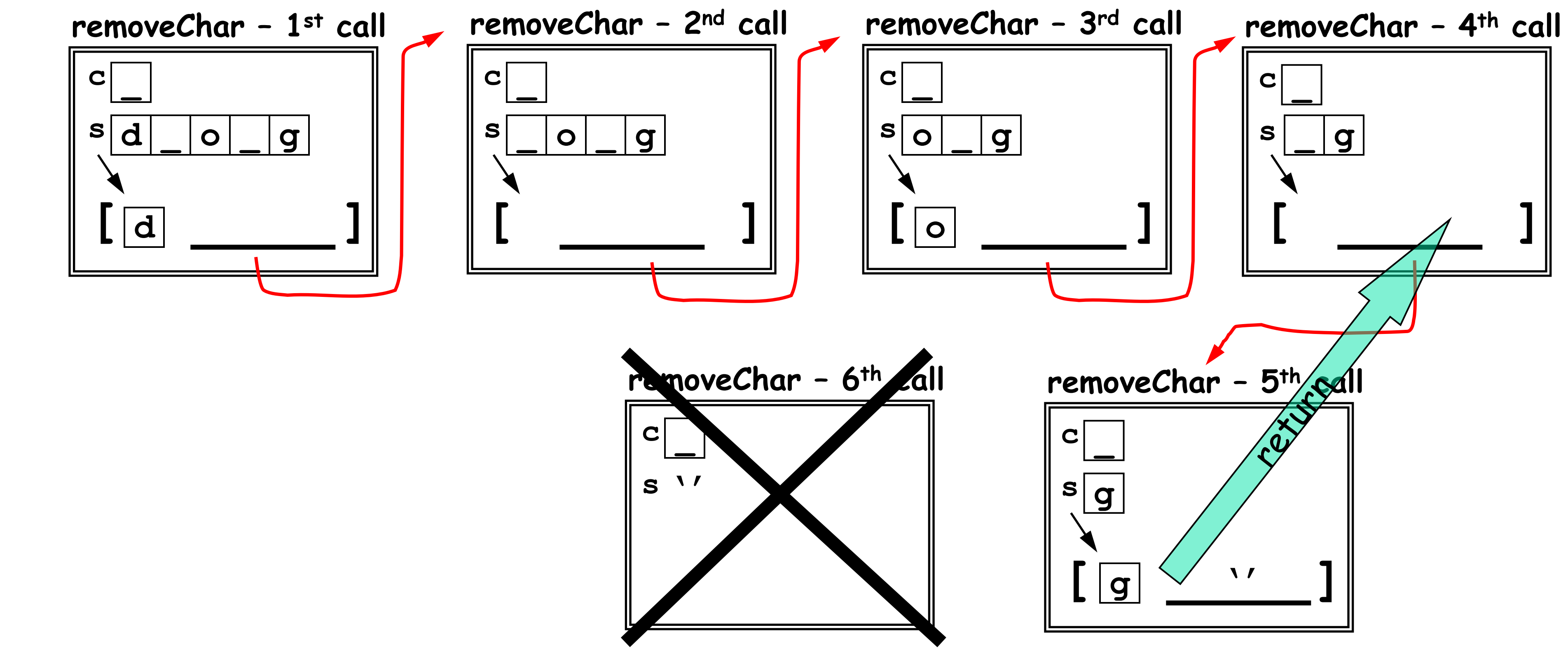

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  1 s= [s(1) removeChar(c, s(2:length(s)))];
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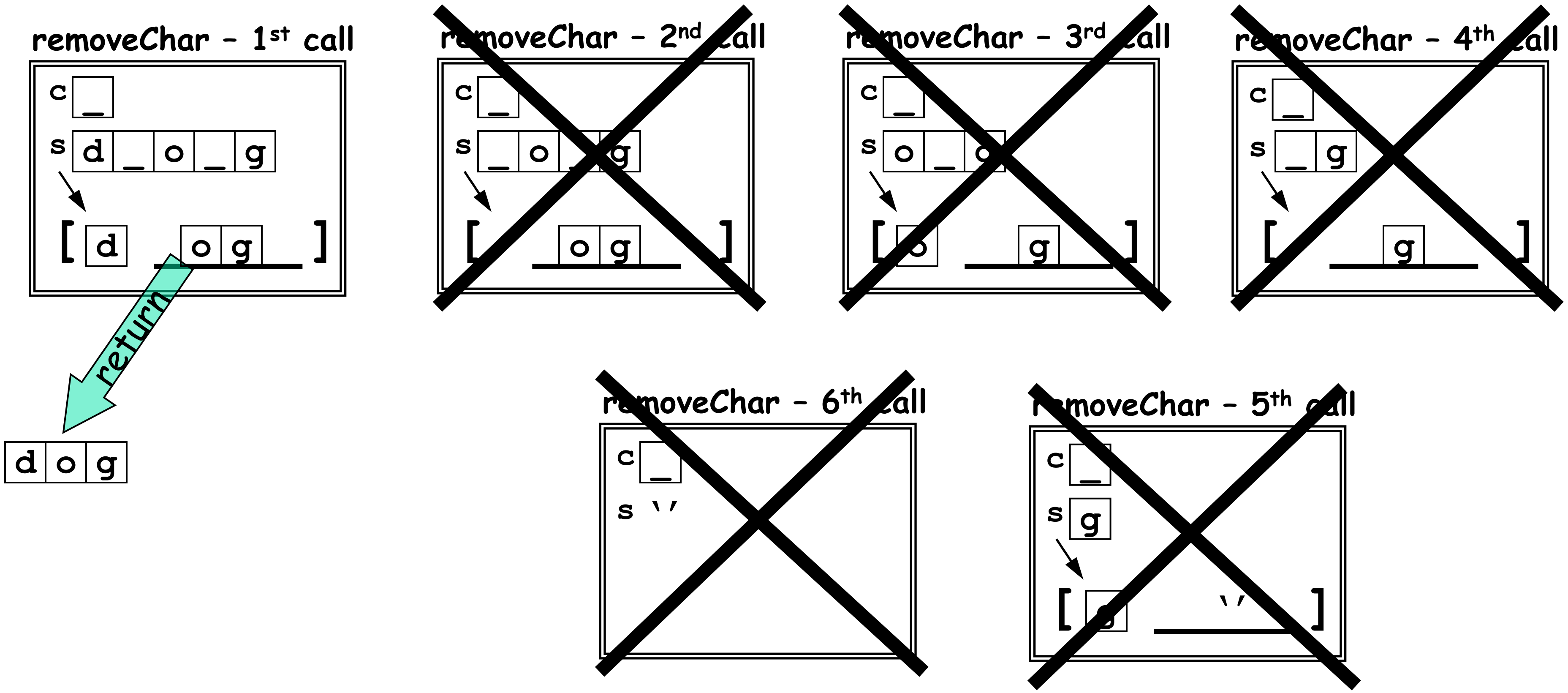
```

removeChar('_', 'd_o_g')



Key to recursion

- Must identify (at least) one base case, the “trivially simple” case
  - no recursion is done in this case
- The recursive case(s) must reflect progress towards the base case
  - E.g., give a shorter vector as the argument to the recursive call – see `removeChar`

```
function s = removeChar(c, s)
if length(s)==0
  return
else
  if s(1)~=c
    s= [s(1) removeChar(c, s(2:length(s)))];
  else
    s= removeChar(c, s(2:length(s)));
  end
end
```

How many call frames are opened (used) in executing each of the following statements?

```
>> st= removeChar('t', 'Matlab');
>> sx= removeChar('x', 'Matlab');
```

A 3, 0

B 4, 1

C 3, 6

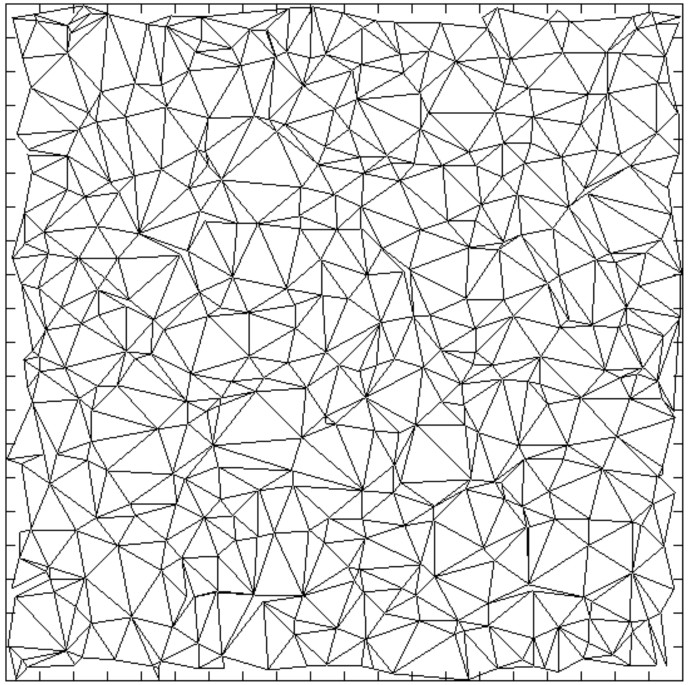
D 6, 6

E 7, 7

Divide-and-conquer methods, such as recursion, is useful in geometric situations

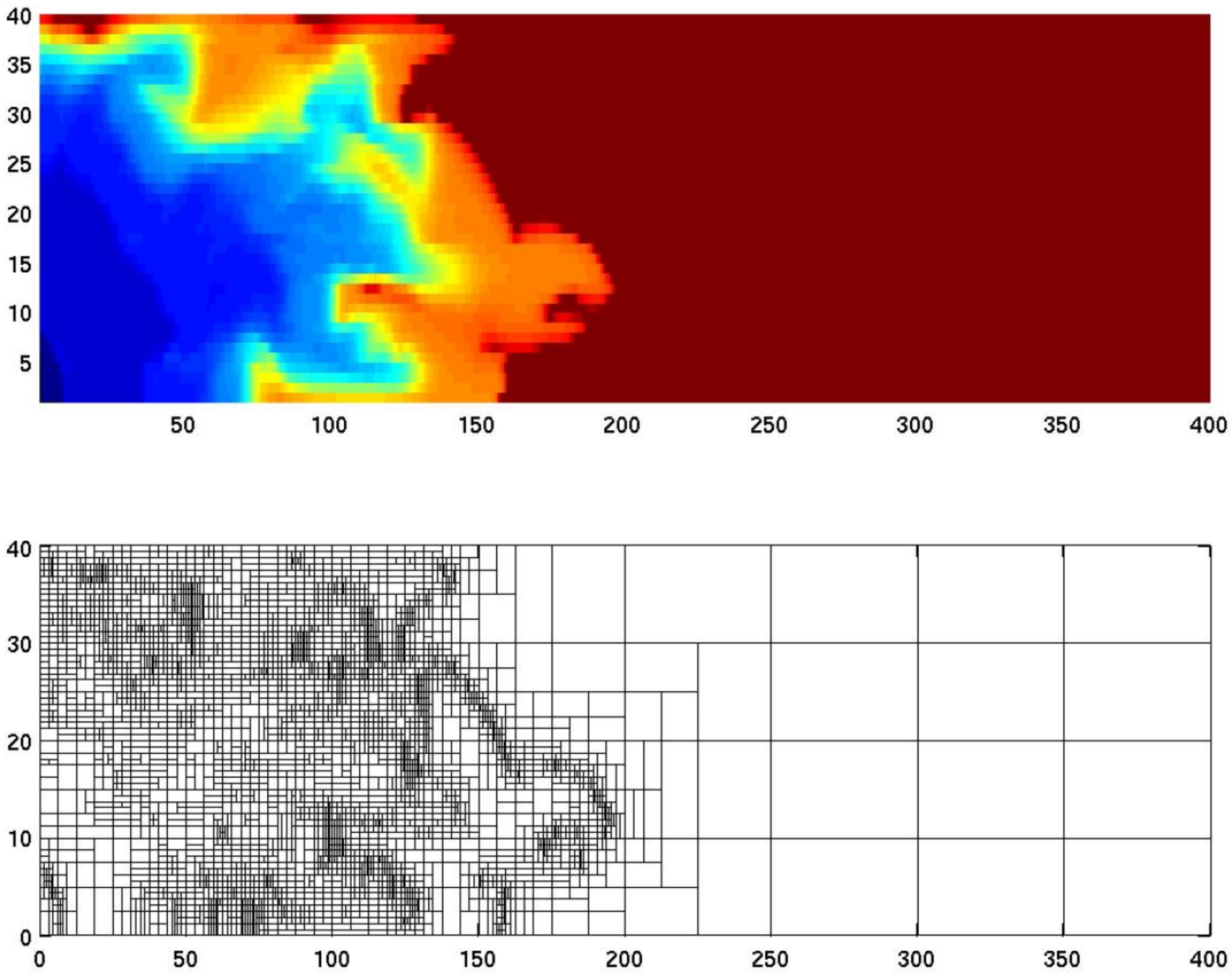
Chop a region up into triangles with smaller triangles in "areas of interest"

3D Graphics: Level of Detail



Recursive mesh generation

Mesh refinement



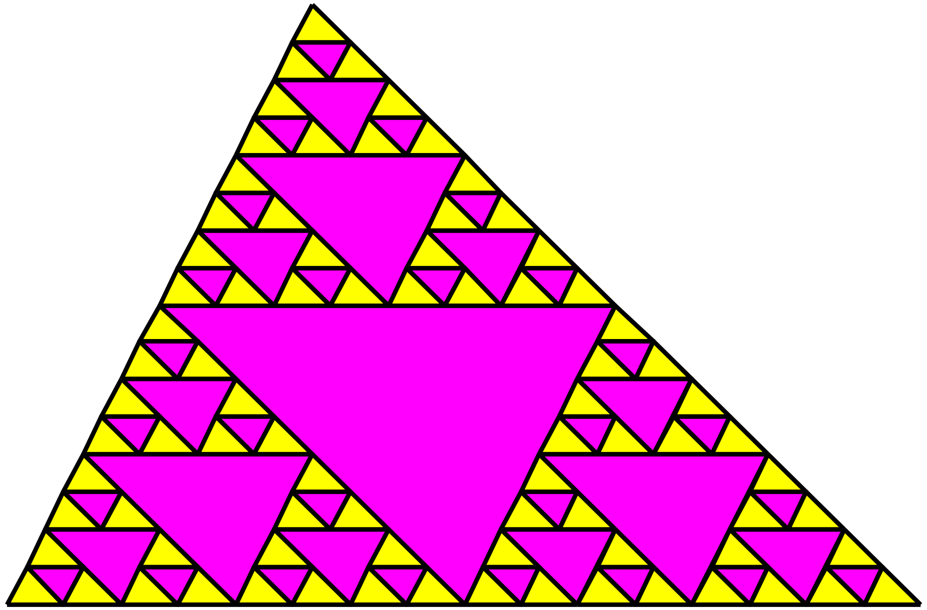
Nilsson, Gerritsen, Younis 2004

When physics is too complicated for one big region, divide it into two smaller regions.

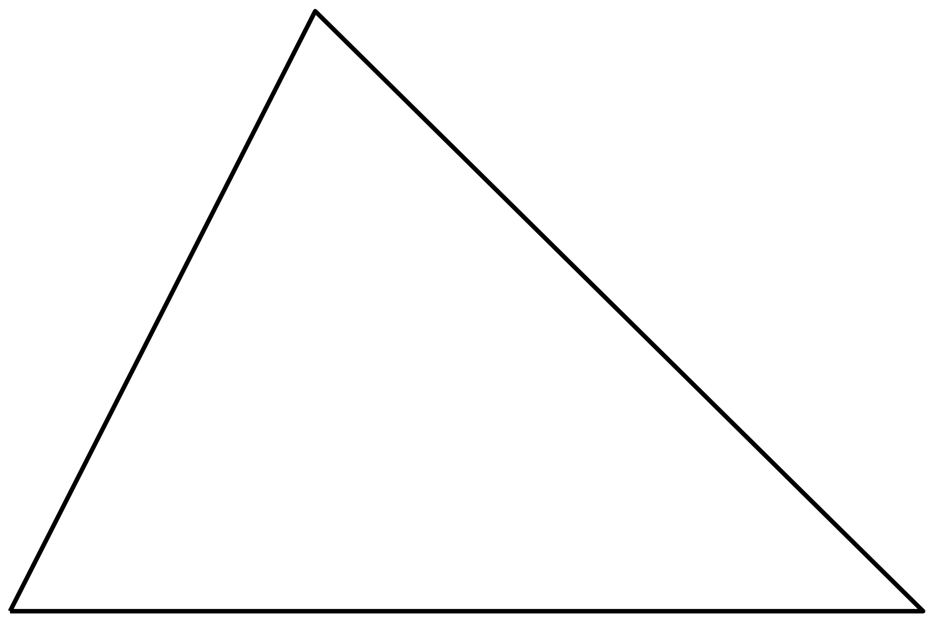
- Subproblem: solve physics inside one region
- Division: split region in half
- Base case: solution looks smooth in entire region

Why is mesh generation a divide-&-conquer process?

Let's draw this graphic

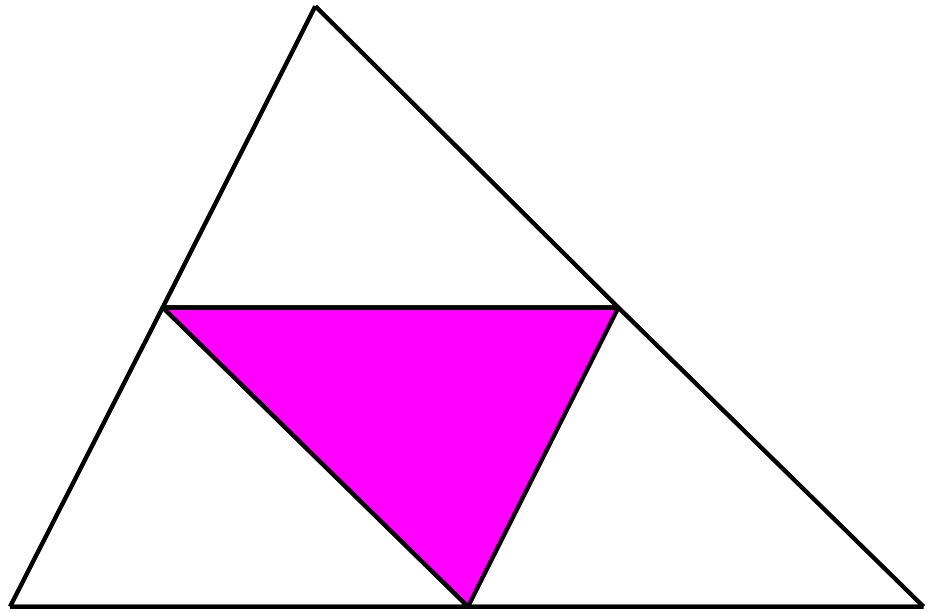


Start with a triangle



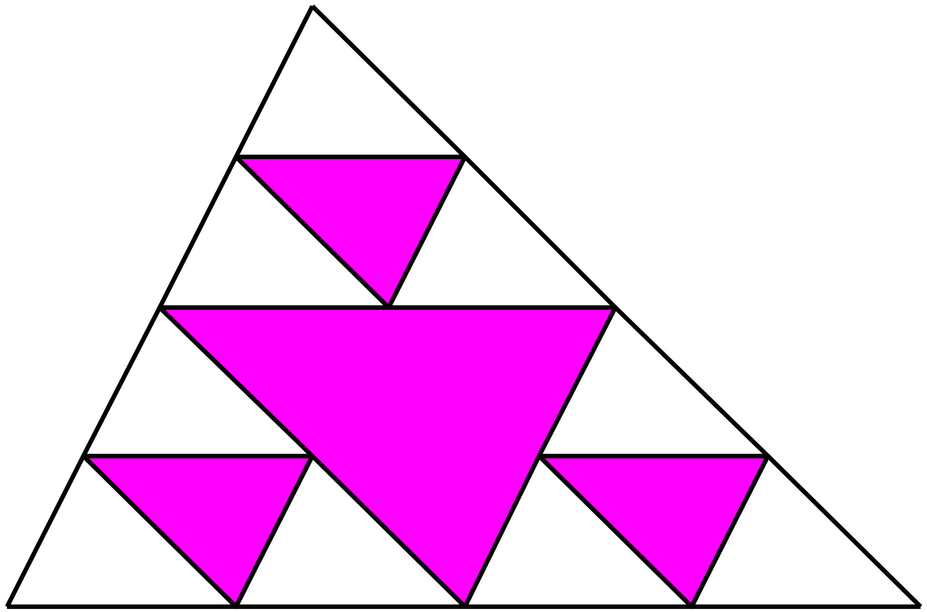
A “level-1” partition of the triangle

(obtained by connecting the midpoints of the sides of the original triangle)

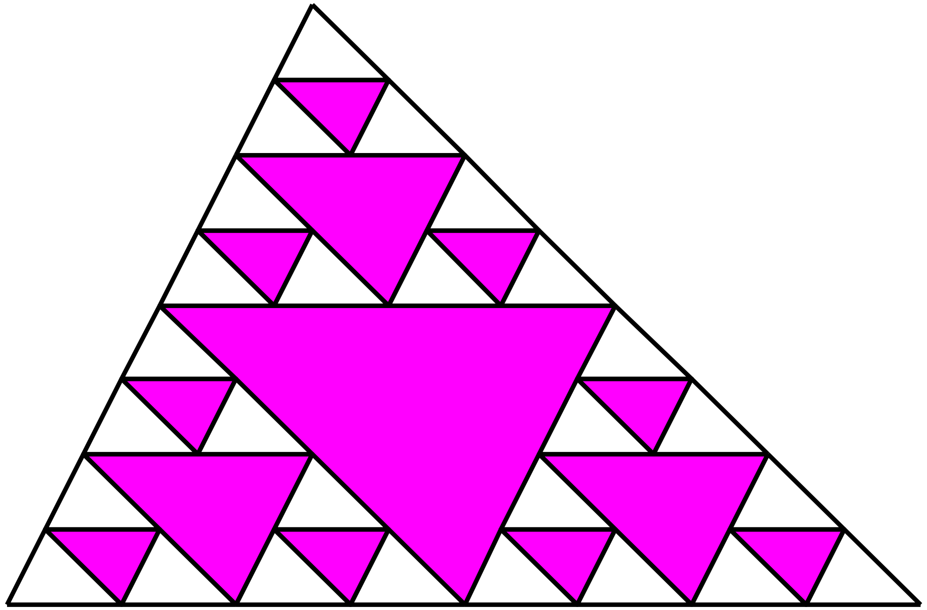


Now do the same partitioning (connecting midpts) on each corner (white) triangle to obtain the “level-2” partitioning

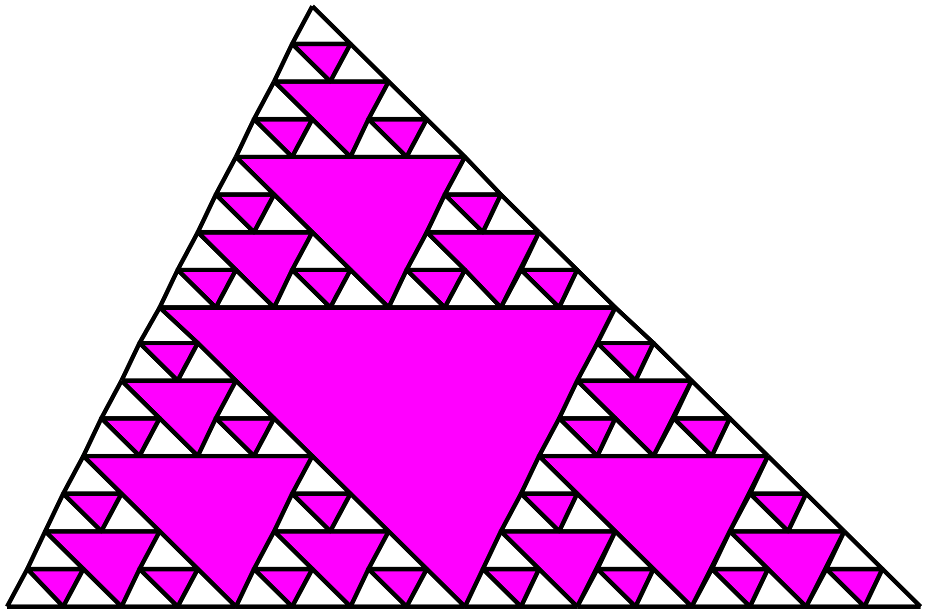
The “level-2” partition of the triangle



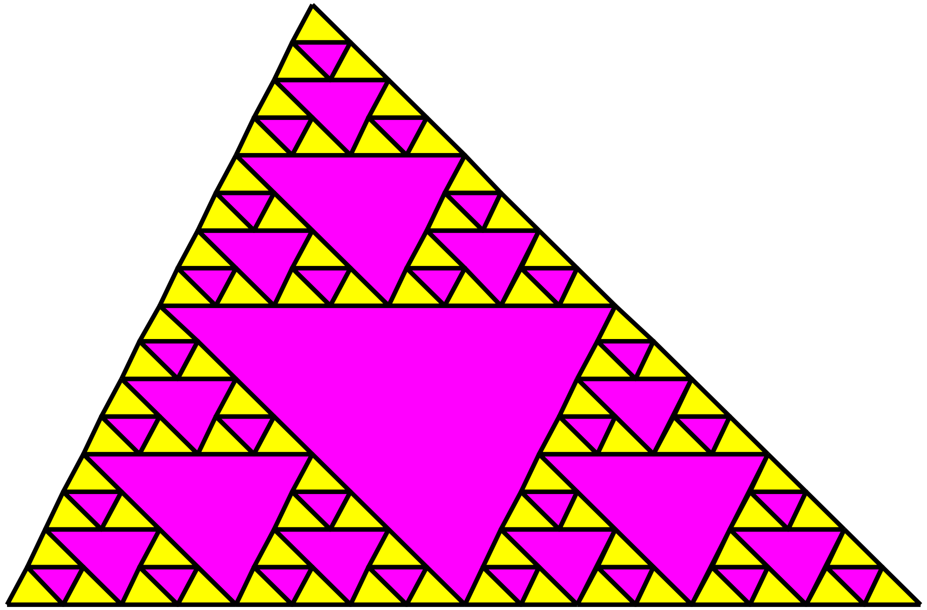
The "level-3" partition of the triangle



The “level-4” partition of the triangle



The “level-4” partition of the triangle



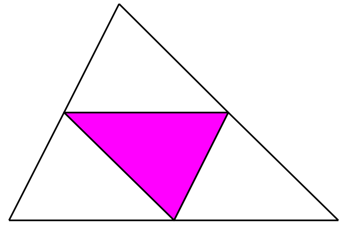
The basic operation at each level

```
if      the triangle is small
        Don't subdivide and just color it yellow.
else
        Subdivide:
        Connect the side midpoints;
        color the interior triangle magenta;
        apply same process to each outer triangle:
                left, right, top;
end
```

```
function MeshTriangle(x,y,L)
% x,y are 3-vectors that define the vertices of a triangle.
% Draw level-L partitioning.  Assume hold is on.

if L==0
  % Recursion limit reached; no more subdivision required.
    fill(x,y,'y')   % Color this triangle yellow

else
  % Need to subdivide:  determine the side midpoints; connect
  % midpts to get “interior triangle”; color it magenta.
```



```
  % Apply the process to the three "corner" triangles...



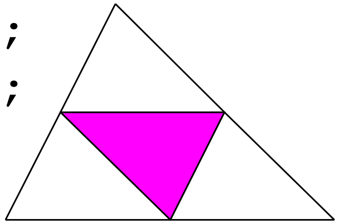
end
```

```
function MeshTriangle(x,y,L)
% x,y are 3-vectors that define the vertices of a triangle.
% Draw level-L partitioning.  Assume hold is on.

if L==0
  % Recursion limit reached; no more subdivision required.
    fill(x,y,'y')   % Color this triangle yellow

else
  % Need to subdivide:  determine the side midpoints; connect
  % midpts to get “interior triangle”; color it magenta.
    a = [(x(1)+x(2))/2 (x(2)+x(3))/2 (x(3)+x(1))/2];
    b = [(y(1)+y(2))/2 (y(2)+y(3))/2 (y(3)+y(1))/2];
    fill(a,b,'m')

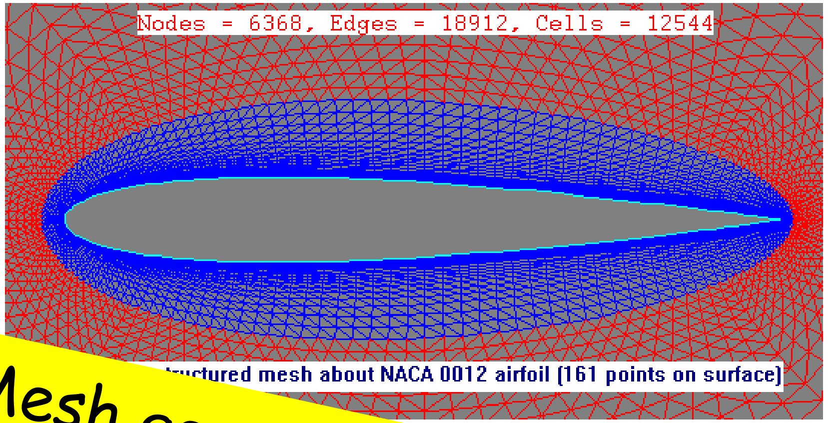
  % Apply the process to the three "corner" triangles...
    MeshTriangle([x(1) a(1) a(3)],[y(1) b(1) b(3)],L-1)
    MeshTriangle([a(1) x(2) a(2)],[b(1) y(2) b(2)],L-1)
    MeshTriangle([a(3) a(2) x(3)],[b(3) b(2) y(3)],L-1)
end
```



Key to recursion

- Must identify (at least) one base case, the "trivially simple" case
  - No recursion is done in this case
- The recursive case(s) must reflect progress towards the base case
  - E.g., give a shorter vector as the argument to the recursive call – see **`removeChar`**
  - E.g., do a lower level of subdivision in the recursive call – see **`MeshTriangle`**

Recursion can be useful in different settings



Mesh generation

Computer graphics

```
                       ┌── exams
                       │
                       ├── exercises
          ┌── cs1112 ──┤              ┌── slides
          │            ├── lecture ───┤
          │            │              └── examples
          │            └── projects
myFiles ──┼── cs1132
          │
          │            ┌── exercises
          └── cs3220 ──┤
                       └── reading
```

Search "tree" structures