A Mathematical Example: Factorial

- Non-recursive definition:

  n! = n × n-1 × … × 2 × 1

  = n (n-1 × … × 2 × 1)

- Recursive definition:

  n! = n (n-1)! for n ≥ 0 **Recursive case**

  0! = 1 **Base case**

What happens if there is no base case?

1

Factorial as a Recursive Function

```
def factorial(n):
    """Returns: factorial of n.
    Pre: n ≥ 0 an int"""
    if n == 0:
        return 1

    return n*factorial(n-1)
```

- n! = n (n-1)!
- 0! = 1

Base case(s)

Recursive case

What happens if there is no base case?

2

Example: Fibonnaci Sequence

- Sequence of numbers: 1, 1, 2, 3, 5, 8, 13, ...

  a_0 a_1 a_2 a_3 a_4 a_5 a_6

  - Get the next number by adding previous two
  - What is a_8?
- Recursive definition:
  - $a_n = a_{n-1} + a_{n-2}$ **Recursive Case**
  - $a_0 = 1$ **Base Case**
  - $a_1 = 1$ **(another) Base Case**

Why did we need two base cases this time?

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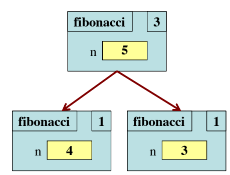
Fibonacci as a Recursive Function

```
def fibonacci(n):
    """Returns: Fibonacci no. an
    Precondition: n ≥ 0 an int"""
    if n <= 1:
        return 1

    return (fibonacci(n-1)+
            fibonacci(n-2))
```

- Function that calls itself
  - Each call is new frame
  - Frames require memory
  - ∞ calls = ∞ memory



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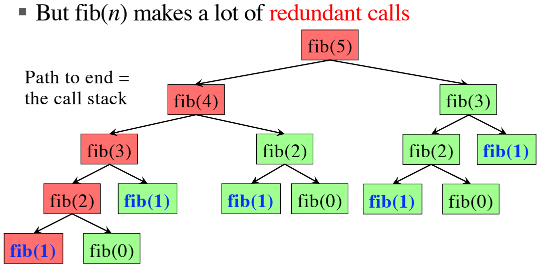
Fibonacci: # of Frames vs. # of Calls

- Fibonacci is very inefficient.
  - fib(n) has a stack that is always ≤ n
  - But fib(n) makes a lot of redundant calls



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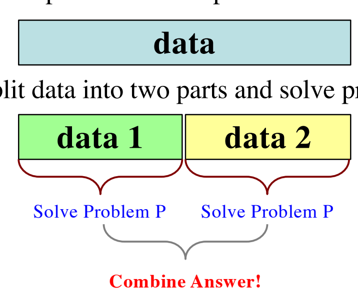
Recursion is best for Divide and Conquer

Goal: Solve problem P on a piece of data

Idea: Split data into two parts and solve problem



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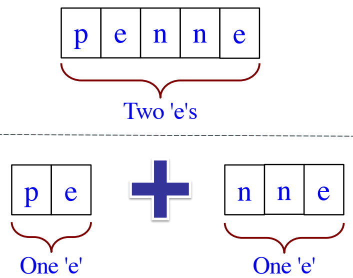
Divide and Conquer Example

Count the number of 'e's in a string:



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Three Steps for Divide and Conquer

1. Decide what to do on "small" data
   - Some data cannot be broken up
   - Have to compute this answer directly
2. Decide how to break up your data
   - Both "halves" should be smaller than whole
   - Often no wrong way to do this (next lecture)
3. Decide how to combine your answers
   - Assume the smaller answers are correct
   - Combining them should give bigger answer

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Divide and Conquer Example

```
def num_es(s):
    """Returns: # of 'e's in s"""
    # 1. Handle small data
    if s == '':
        return 0
    elif len(s) == 1:
        return 1 if s[0] == 'e' else 0

    # 2. Break into two parts
    left = num_es(s[0])
    right = num_es(s[1:])

    # 3. Combine the result
    return left+right
```

"Short-cut" for

```
if s[0] == 'e':
    return 1
else:
    return 0
```

s[0] s[1:]

p | e n n e

0 + 2

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Exercise: Remove Blanks from a String

```
def deblank(s):
    """Returns: s w/o blanks"""
    if s == '':
        return s
    elif len(s) == 1:
        return '' if s[0] == ' ' else s

    left = deblank(s[0])
    right = deblank(s[1:])

    return left+right
```

Handle small data

Break up the data

Combine answers

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Minor Optimization

```
def deblank(s):
    """Returns: s w/o blanks"""
    if s == '':
        return s

    left = s[0]
    if s[0] == ' ':
        left = ''
    right = deblank(s[1:])

    return left+right
```

Eliminate the second base by combining

Less recursive calls

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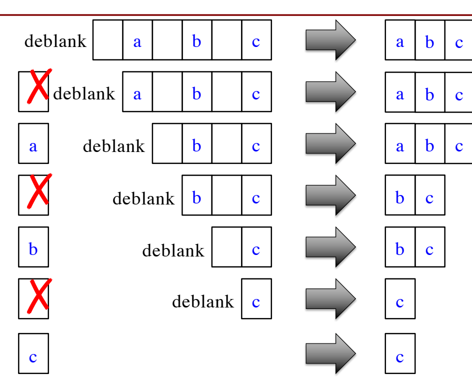
Following the Recursion



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