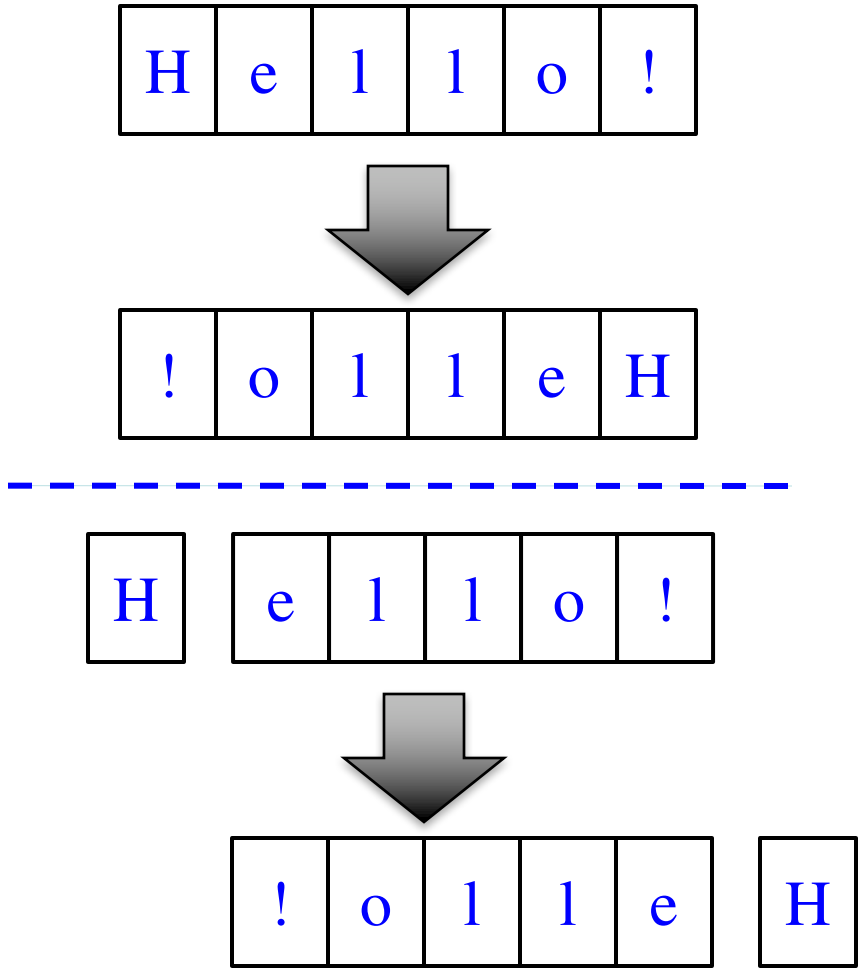
Example: Reversing a String

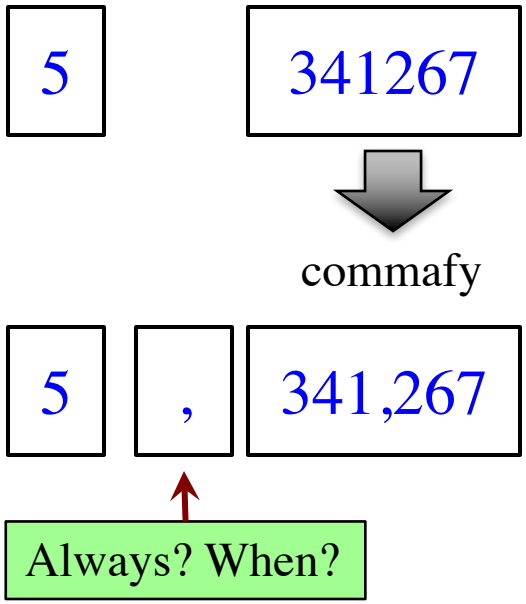
```
def reverse(s):
    """Returns: reverse of s

    Precondition: s a string"""
    # 1. Handle small data
    if len(s) <= 1:
        return s


    # 2. Break into two parts
    left  = s[0]
    right = reverse(s[1:])

    # 3. Combine the result
    return right+left
```



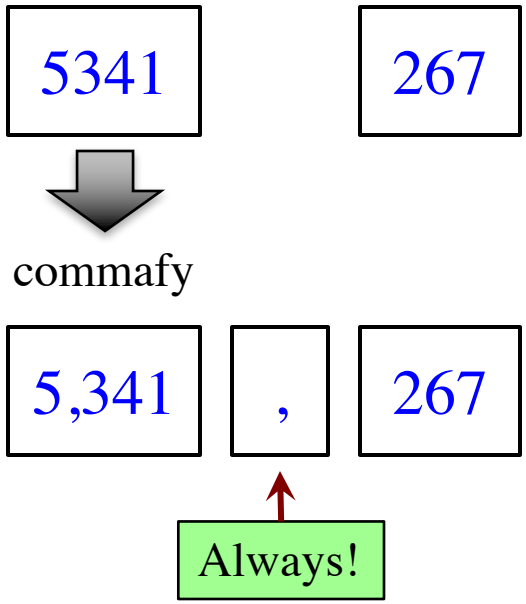
How to Break Up a Recursive Function?

```
def commafy(s):
    """Returns: string with commas every 3 digits
    e.g. commafy('5341267') = '5,341,267'
    Precondition: s represents a non-negative int"""
```

Approach 1



Approach 2



How to Break Up a Recursive Function?

```
def commafy(s):
    """Returns: string with commas every 3 digits
    e.g. commafy('5341267') = '5,341,267'
    Precondition: s represents a non-negative int"""
    # 1. Handle small data.
    if len(s) <= 3:
        return s

    # 2. Break into two parts
    left  = commafy(s[:-3])
    right = s[-3:]  # Small part on RIGHT

    # 3. Combine the result
    return left + ',' + right
```

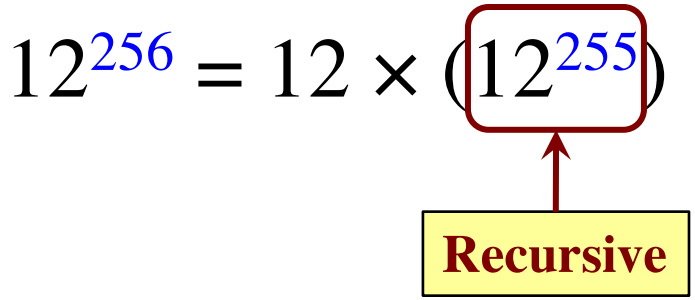
Base Case

Recursive Case

How to Break Up a Recursive Function?

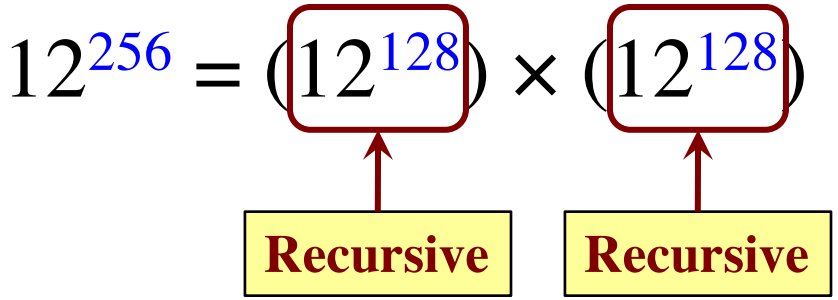
```
def exp(b, c)
    """Returns: b^c
    Precondition: b a float, c ≥ 0 an int"""
```

Approach 1



$b^c = b \times (b^{c-1})$

Approach 2



$b^c = (b \times b)^{c/2}$ if c even

Raising a Number to an Exponent

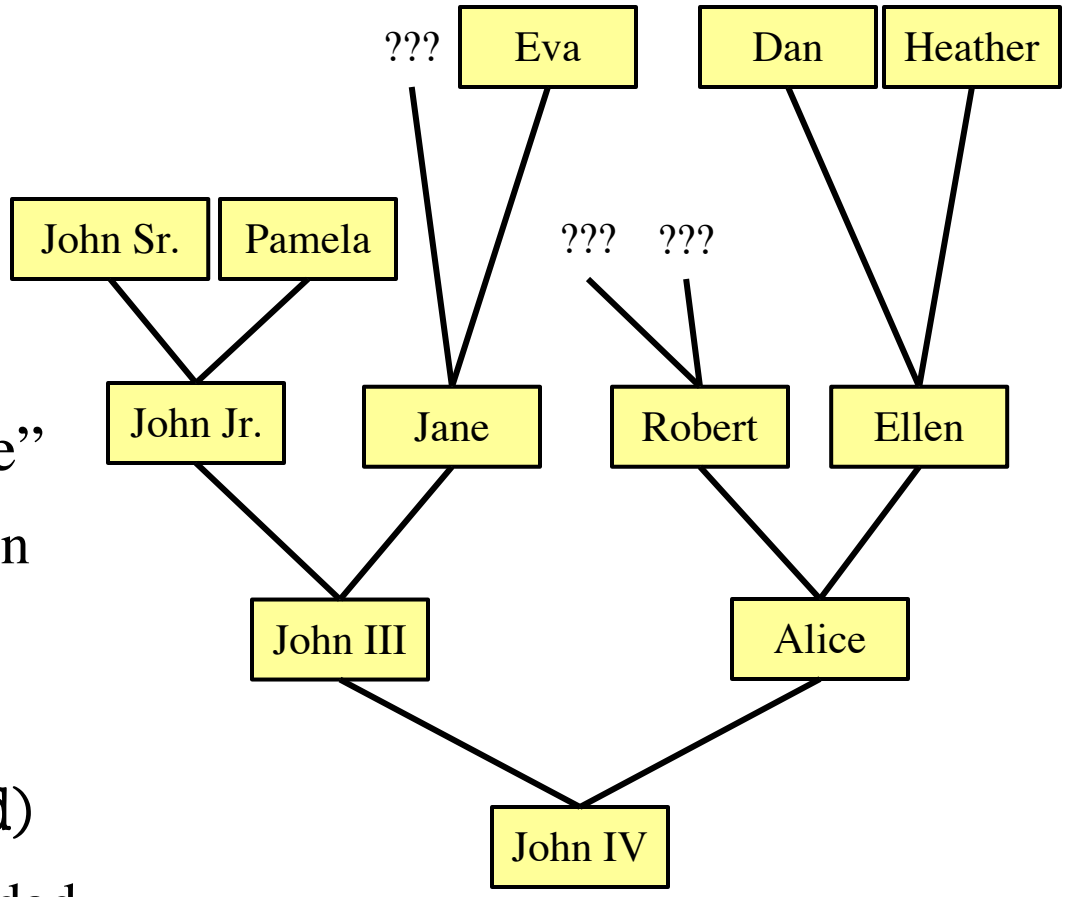
```
def exp(b, c)
    """Returns: b^c
    Precond: b a float, c ≥ 0 an int"""
    # b^0 is 1
    if c == 0:
        return 1

    # c > 0
    if c % 2 == 0:
        return exp(b*b,c/2)

    return b*exp(b*b,(c-1)/2)
```

c	# of calls
0	0
1	1
2	2
4	3
8	4
16	5
32	6
2^n	n + 1

32768 is 215
b^{32768} needs only 215 calls!

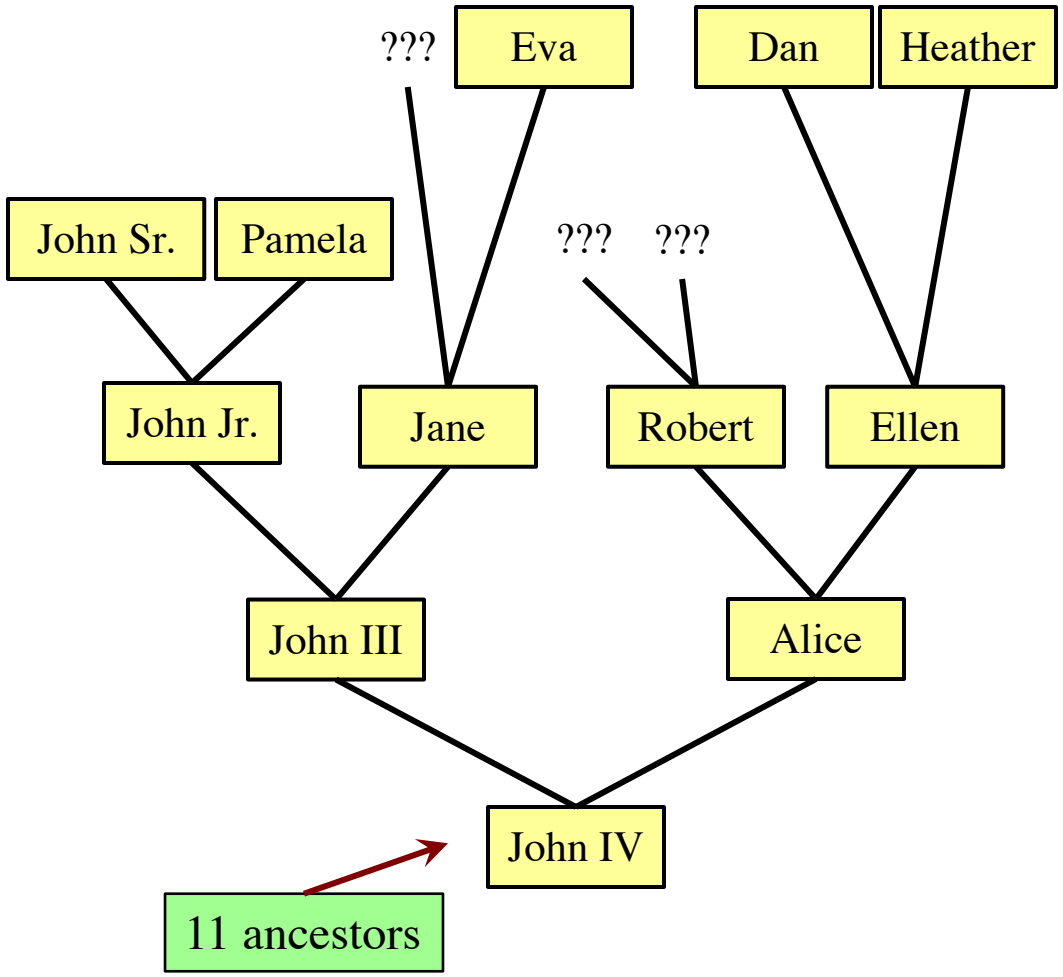
Recursion and Objects

- Class Person (`person.py`)
  - Objects have 3 attributes
  - `name`: String
  - `mom`: Person (or `None`)
  - `dad`: Person (or `None`)
- Represents the "family tree"
  - Goes as far back as known
  - Attributes `mom` and `dad` are `None` if not known
- **Constructor**: `Person(n,m,d)`
  - Or `Person(n)` if no mom, dad



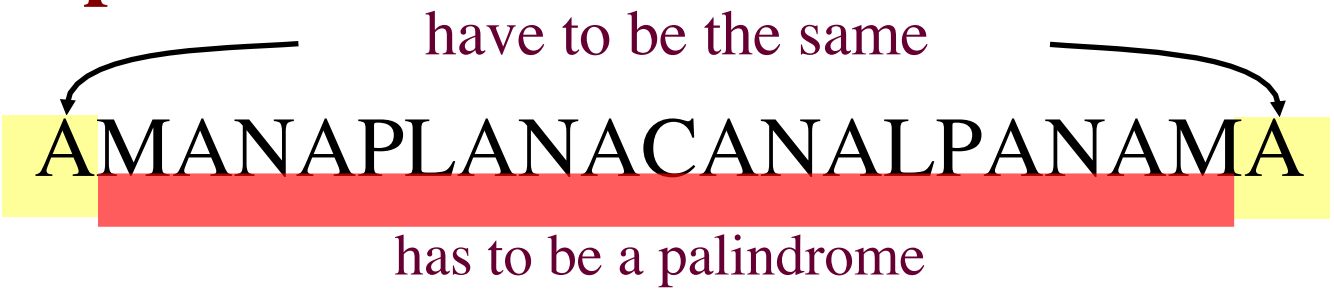
Recursion and Objects

```
def num_ancestors(p):
    """Returns: num of known ancestors
    Pre: p is a Person"""
    # 1. Handle small data.
    if p.mom == None and p.dad == None:
        return 0

    # 2. Break into two parts
    moms = 0
    if not p.mom == None:
        moms = 1+num_ancestors(p.mom)
    dads = 0
    if not p.dad== None:
        dads = 1+num_ancestors(p.dad)

    # 3. Combine the result
    return moms+dads
```



Example: Palindromes

- String with ≥ 2 characters is a palindrome if:
  - its first and last characters are equal, and
  - the rest of the characters form a palindrome
- **Example:**



- **Function to Implement:**

```
def ispalindrome(s):
    """Returns: True if s is a palindrome"""
```

Example: Palindromes

- String with ≥ 2 characters is a palindrome if:
  - its first and last characters are equal, and
  - the rest of the characters form a palindrome

Recursive Definition

```
def ispalindrome(s):
    """Returns: True if s is a palindrome"""
    if len(s) < 2:
        return True

    # Halves not the same; not divide and conquer
    ends = s[0] == s[-1]
    middle = ispalindrome(s[1:-1])
    return ends and middle
```

Base case

Recursive case

Recursive Functions and Helpers

```
def ispalindrome2(s):
    """Returns: True if s is a palindrome
    Case of characters is ignored."""
    if len(s) < 2:
        return True
    # Halves not the same; not divide and conquer
    ends = equals_ignore_case(s[0], s[-1])
    middle = ispalindrome(s[1:-1])
    return ends and middle


def equals_ignore_case(a, b):
    """Returns: True if a and b are same ignoring case"""
    return a.upper() == b.upper()
```

Use helper functions!

- Pull out anything not part of the recursion
- Keeps your code simple and easy to follow

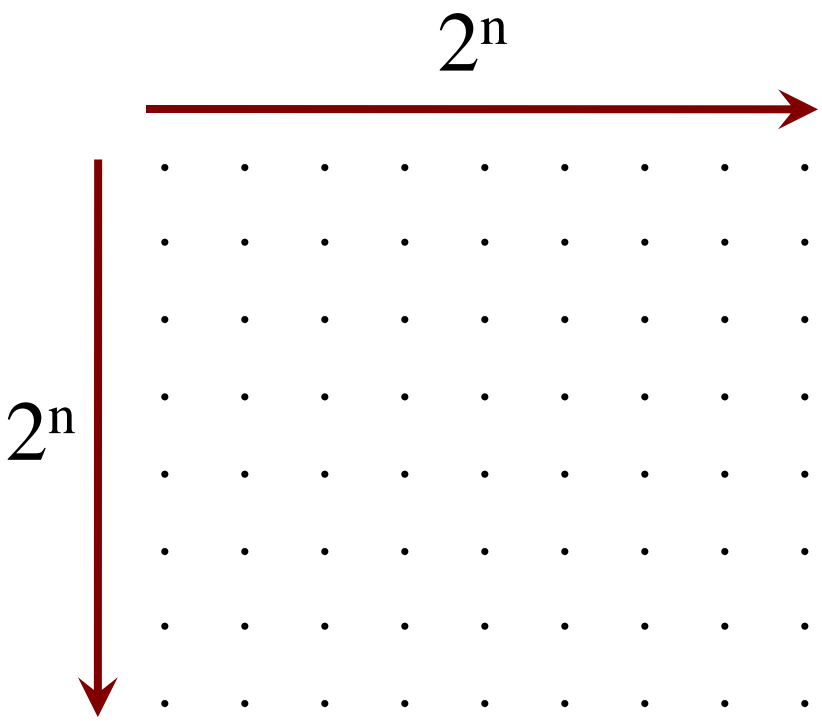
Example: More Palindromes

```
def ispalindrome3(s):
    """Returns: True if s is a palindrome
    Case of characters and non-letters ignored."""
    return ispalindrome2(depunct(s))


def depunct(s):
    """Returns: s with non-letters removed"""
    if s == '':
        return s
    # Combine left and right
    if s[0] in string.letters:
        return s[0]+depunct(s[1:])
    # Ignore left if it is not a letter
    return depunct(s[1:])
```

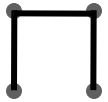
Use helper functions!

- Sometimes the helper is a recursive function
- Allows you break up problem in smaller parts

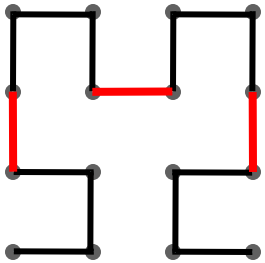
Hilbert's Space Filling Curve



Hilbert(1):



Hilbert(2):



Hilbert(n):