Lecture 25

# Sequence Algorithms (Continued)

# Announcements for This Lecture

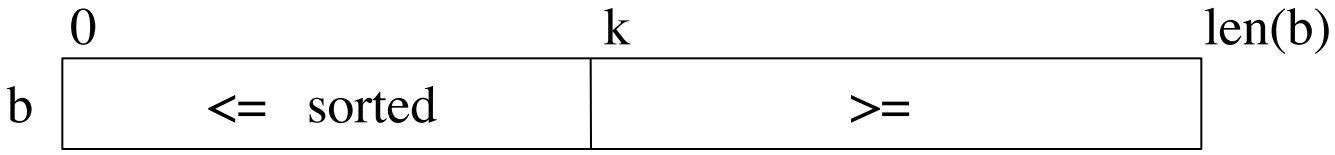
## Assignment & Lab

- A6 is not graded yet
  - Done by end of classes
- A7 due **Wed, Dec. 10**
  - Wednesday after classes
  - Keep on top of milestones
  - Is your paddle moving?
- Lab Today: Office Hours
  - Get help on A7 paddle
  - Anyone can go to any lab

## Next Week

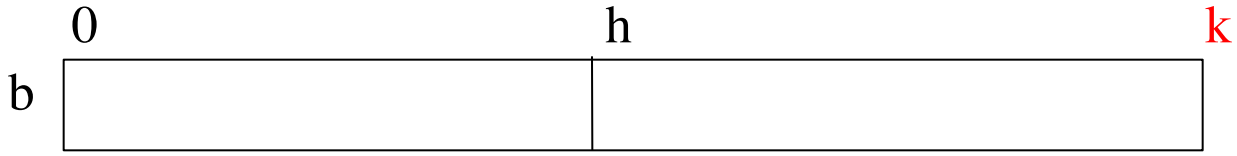
- Last Week of Class!
  - Finish sorting algorithms
  - Special final lecture
- Lab held, but is optional
  - More invariant practice
  - Also use lab time on A7
- Details about the exam
  - Multiple review sessions

# Recall: Horizontal Notation



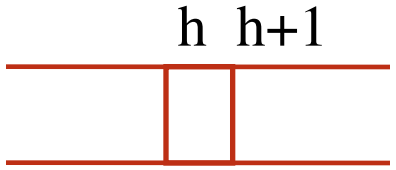

Example of an assertion about an sequence b. It asserts that:

1. b[0..k–1] is sorted (i.e. its values are in ascending order)
2. Everything in b[0..k–1] is ≤ everything in b[k..len(b)–1]




Given index h of the first element of a segment and index k of the element that follows that segment, the number of values in the segment is k – h.

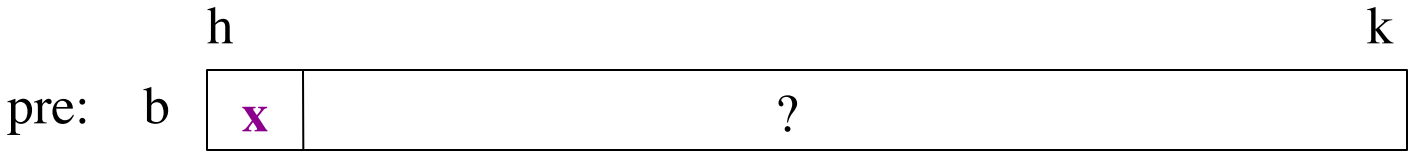
b[h .. k – 1] has k – h elements in it.



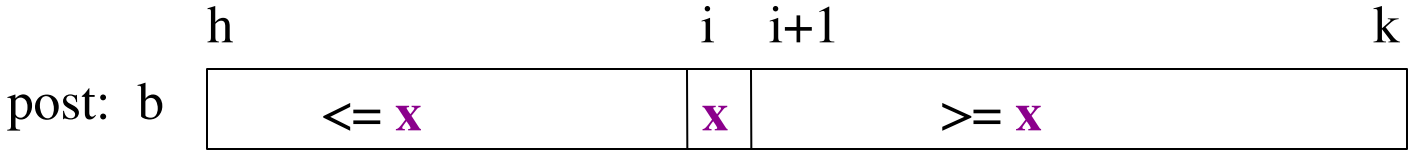

(h+1) – h = 1

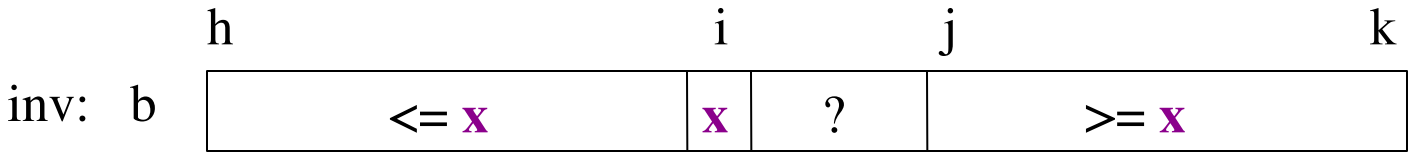
# Partition Algorithm

- Given a sequence b[h..k] with some value x in b[h]:




- Swap elements of b[h..k] and store in j to truthify post:




- Agrees with precondition when i = h, j = k+1
- Agrees with postcondition when j = i+1

# Partition Algorithm Implementation

```
def partition(b, h, k):
    """Partition list b[h..k] around a pivot x = b[h]"""
    i = h; j = k+1; x = b[h]
    # invariant: b[h..i-1] < x, b[i] = x, b[j..k] >= x
    while i < j-1:
        if b[i+1] >= x:
            # Move to end of block.
            _swap(b,i+1,j-1)
            j = j - 1
        else:   # b[i+1] < x
            _swap(b,i,i+1)
            i = i + 1
    # post: b[h..i-1] < x, b[i] is x, and b[i+1..k] >= x
    return i
```

**partition(b,h,k), not partition(b[h:k+1])**

Remember, slicing always copies the list!

We want to partition the **original** list

# Partition Algorithm Implementation

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    return i
```

| <= x | x | ? | >= x |
|---|---|---|---|
| h | i | i+1 | j k |
| 1 2 | 3 | 1 5 0 | 6 3 8 |

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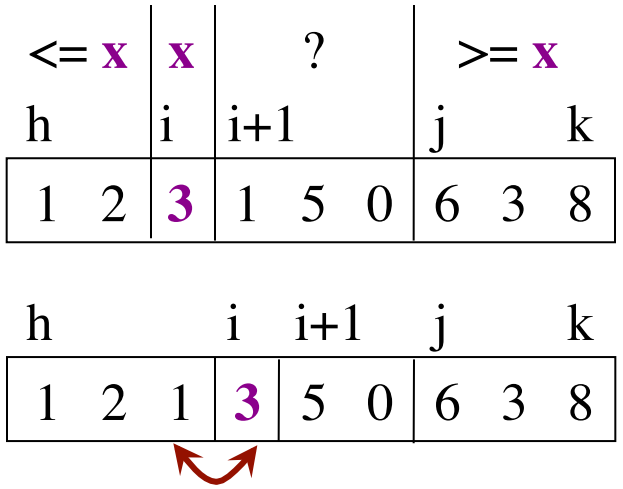

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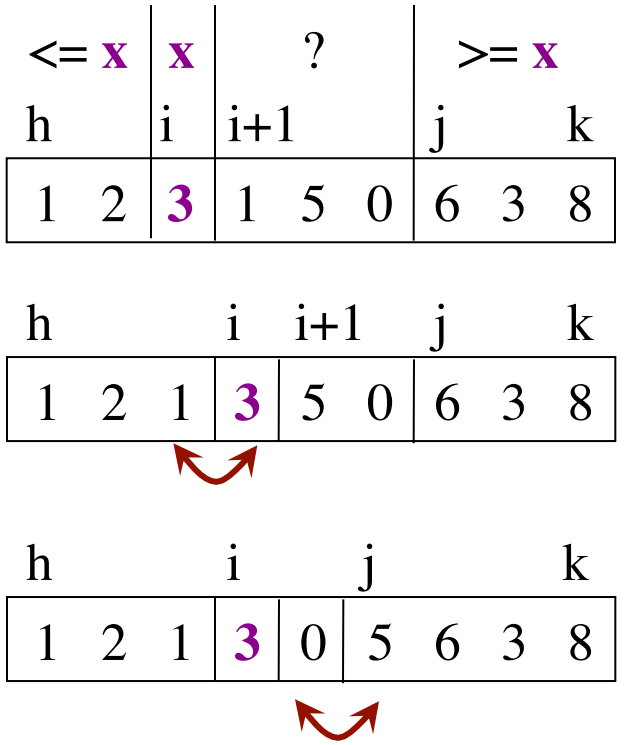

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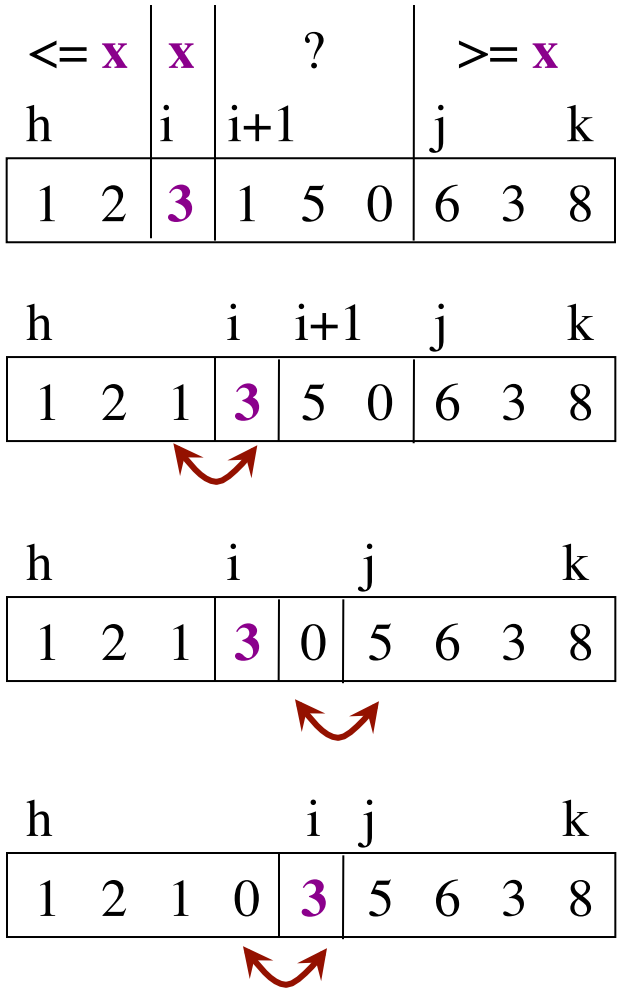
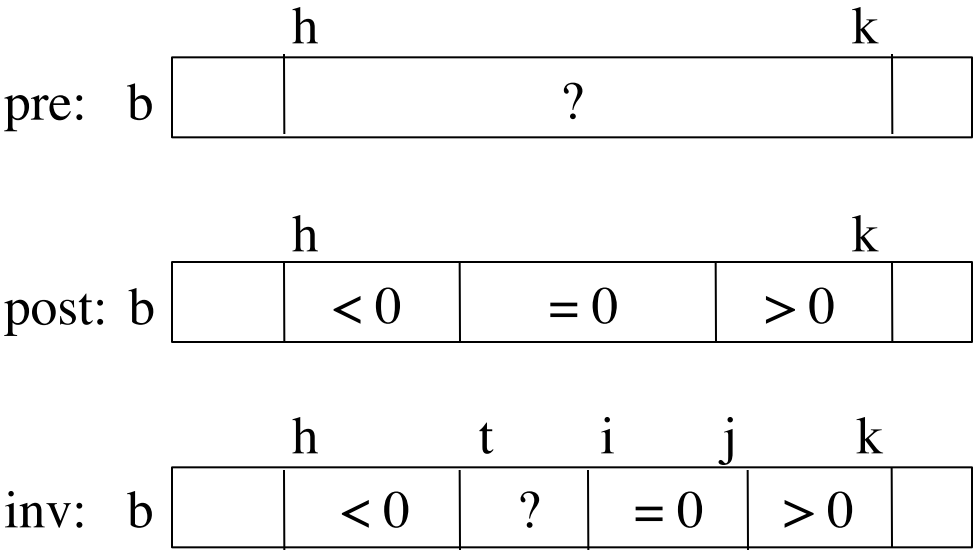


# Dutch National Flag Variant

- Sequence of integer values
  - ‘red’ = negatives, ‘white’ = 0, ‘blues’ = positive
  - Only rearrange part of the list, not all



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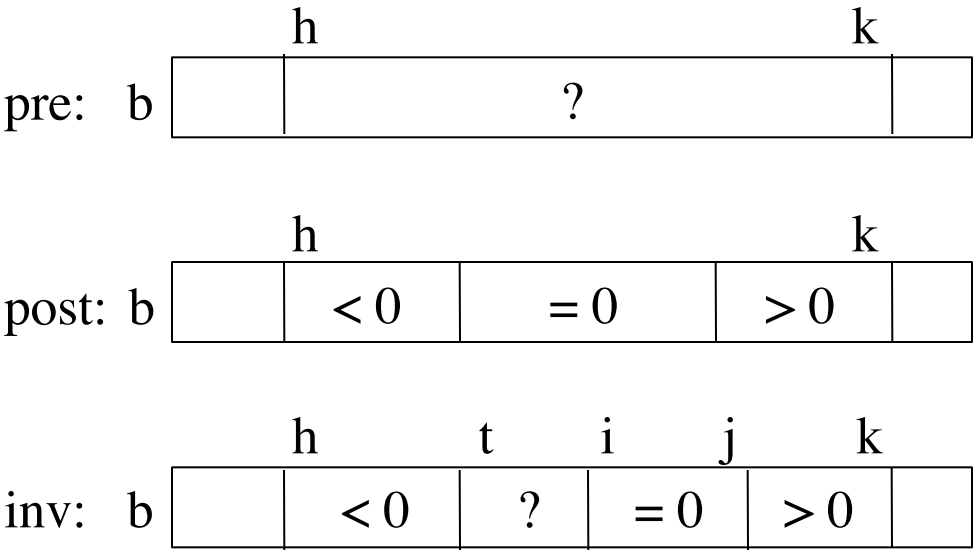


**pre**: t = h,
i = k+1,
j = k
**post**: t = i

# Dutch National Flag Algorithm

```
def dnf(b, h, k):
    """Returns: partition points as a tuple (i,j)"""
    t = h; i = k+1, j = k;
    # inv: b[h..t-1] < 0, b[t..i-1] ?, b[i..j] = 0, b[j+1..k] > 0
    while t < i:
        if b[i-1] < 0:
            swap(b,i-1,t)
            t = t+1
        elif b[i-1] == 0:
            i = i-1
        else:
            swap(b,i-1,j)
            i = i-1; j = j-1
    # post: b[h..i-1] < 0, b[i..j] = 0, b[j+1..k] > 0
    return (i, j)
```

| < 0 | | ? | | | = 0 | | > 0 | |
|---|---|---|---|---|---|---|---|---|
| h | | t | | | i | j | | k |
| -1 | -2 | 3 | -1 | 0 | 0 | 0 | 6 | 3 |

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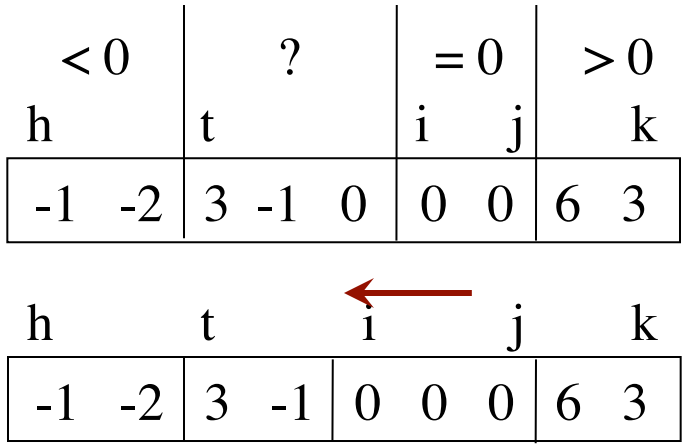

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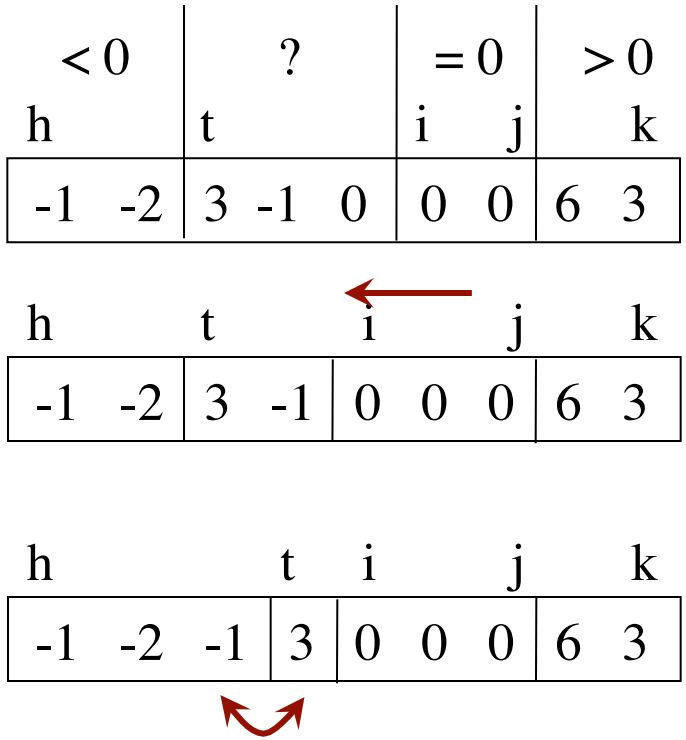

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    while t < i:
        if b[i-1] < 0:
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            t = t+1
        elif b[i-1] == 0:
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        else:
            swap(b,i-1,j)
            i = i-1; j = j-1
    # post: b[h..i-1] < 0, b[i..j] = 0, b[j+1..k] > 0
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```

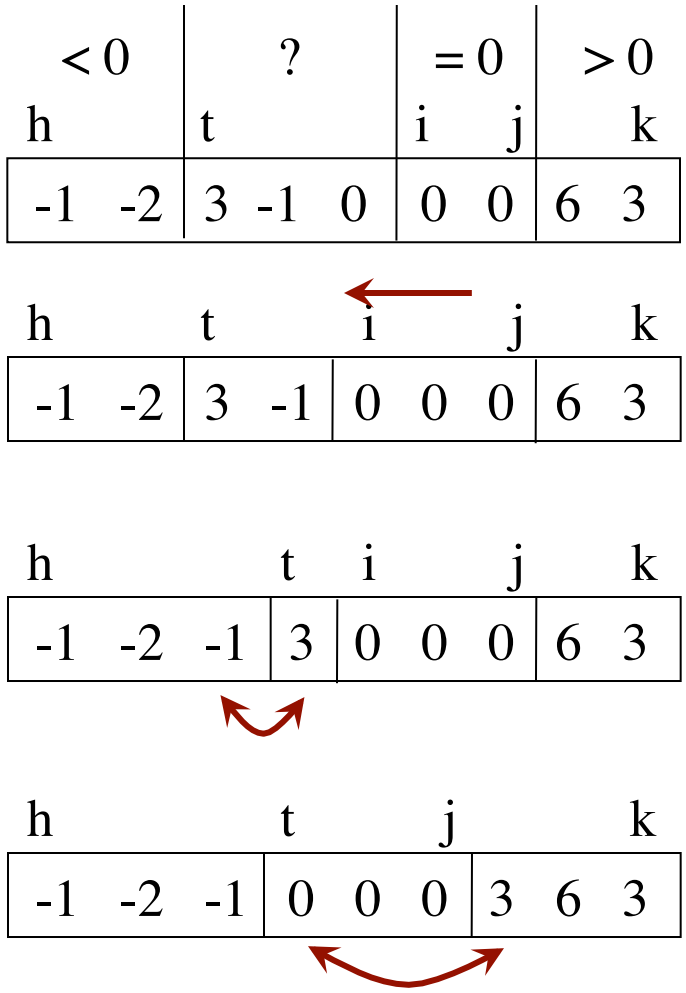

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- **Better**: Store an integer in i to truthify result condition post:

  post: 1. v is not in b[h..i-1]

  2. i = k OR v = b[i]

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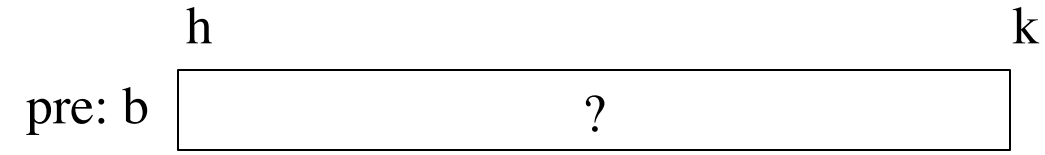


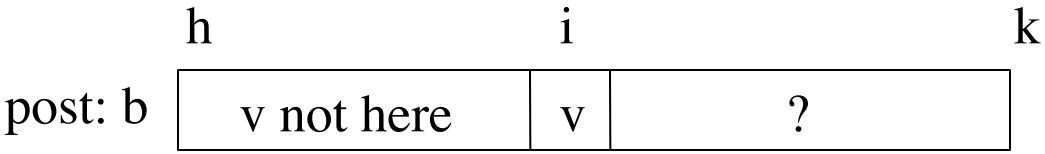

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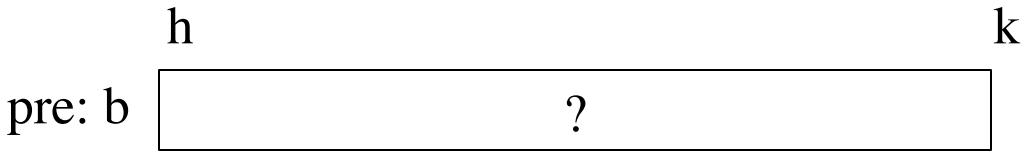
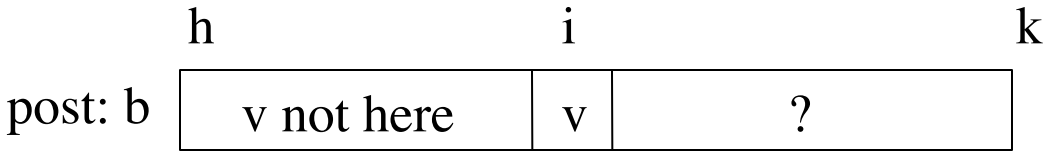
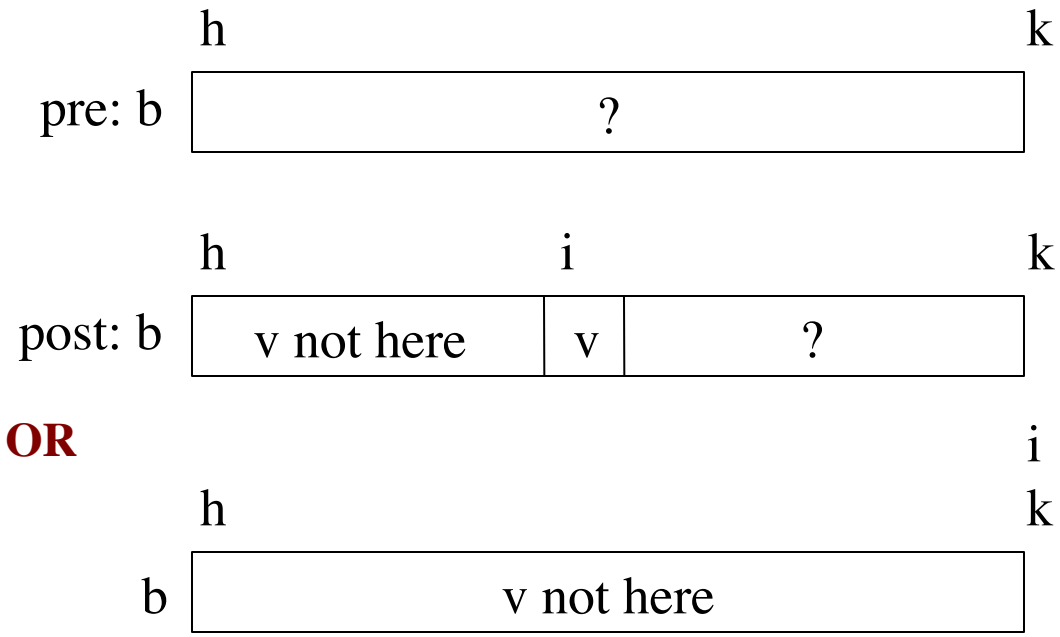
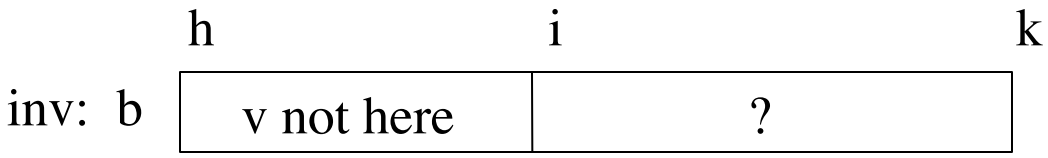



**OR**

# Linear Search



# Linear Search

```
def linear_search(b,c,h):
    """Returns: first occurrence of c in b[h..]"""
    # Store in i the index of the first c in b[h..]
    i = h

    # invariant: c is not in b[0..i-1]
    while i < len(b) and b[i] != c:
        i = i + 1

    # post: c is not in b[h..i-1]
    #           i >= len(b) or b[i] == c
    return i if i < len(b) else -1
```

## Analyzing the Loop

1. Does the initialization make **inv** true?

2. Is **post** true when **inv** is true and **condition** is false?

3. Does the repetend make progress?

4. Does the repetend keep the invariant **inv** true?

# Binary Search

- **Vague:** Look for v in **sorted** sequence segment b[h..k].

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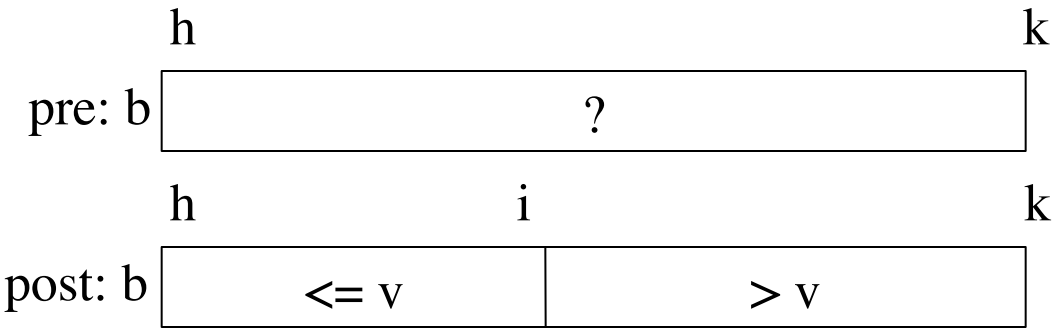
- **Better:**
  - Precondition: b[h..k-1] is sorted (in ascending order).
  - Postcondition: b[h..i] <= v and v < b[i+1..k-1]
- Below, the array is in non-descending order:



# Binary Search

- **Vague:** Look for v in **sorted** sequence segment b[h..k].
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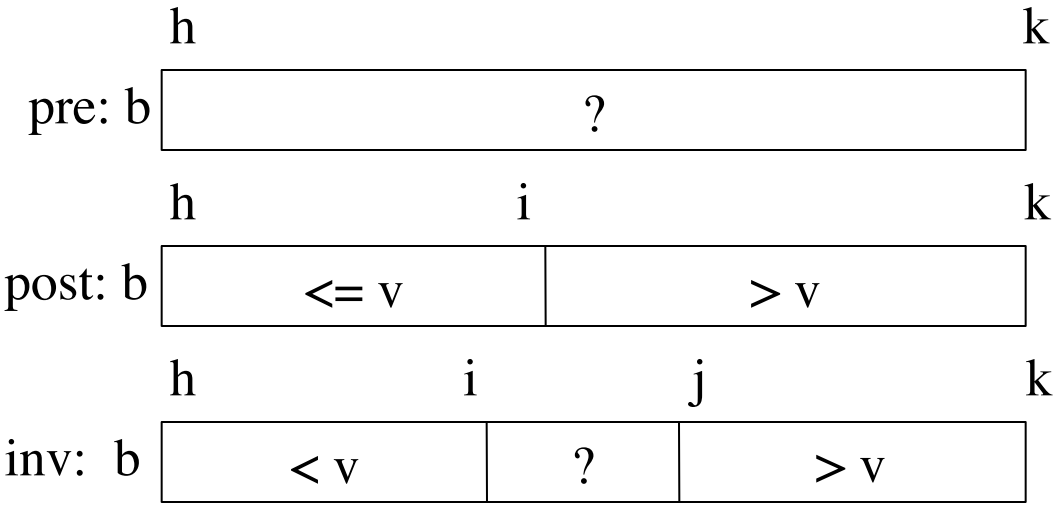



Called binary search because each iteration of the loop cuts the array segment still to be processed in half

# Extras Not Covered in Class

# Loaded Dice

- Sequence p of length n represents n-sided die
  - Contents of p sum to 1
  - p[k] is probability die rolls the number k

| 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 0.1 | 0.1 | 0.1 | 0.1 | 0.3 | 0.3 |

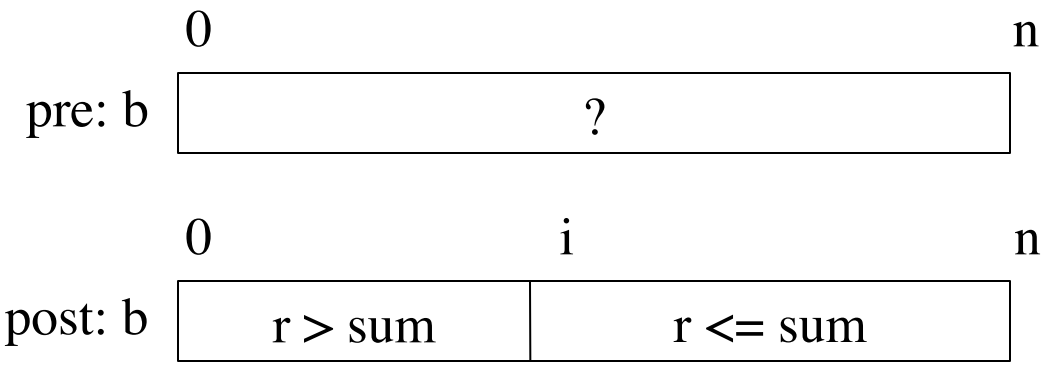
weighted d6, favoring 5, 6

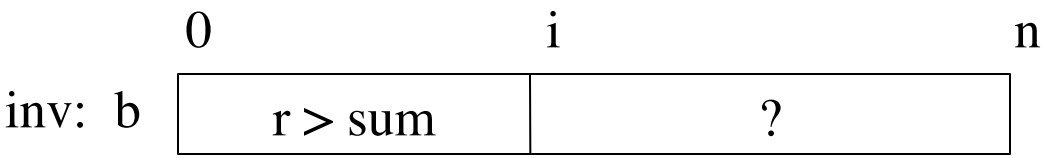
- Goal: Want to "roll the die"
  - Generate random number r between 0 and 1
  - Pick p[i] such that $p[i-1] < r \leq p[i]$

| 0.1 | 0.1 | 0.1 | 0.1 | 0.3 | 0.3 |
|---|---|---|---|---|---|
| 0.1 | 0.2 | 0.3 | 0.4 | 0.7 | 1.0 |

# Loaded Dice

- **Want**: Value i such that p[i-1] < r <= p[i]




- Same as precondition if i = 0
- Postcondition is invariant + false loop condition

# Loaded Dice

```
def roll(p):
    """Returns: randint in 0..len(p)-1; i returned with prob. p[i]
    Precondition: p list of positive floats that sum to 1."""
    r = random.random()     # r in [0,1)
    # Think of interval [0,1] divided into segments of size p[i]
    # Store into i the segment number in which r falls.
    i = 0;    sum_of = p[0]
    # inv: r >= sum of p[0] .. p[i-1]; pEnd = sum of p[0] .. p[i]
    while r >= sum_of:
        sum_of = sum_of + p[i+1]
        i = i + 1

    # post: sum of p[0] .. p[i-1] <= r < sum of p[0] .. p[i]
    return i
```

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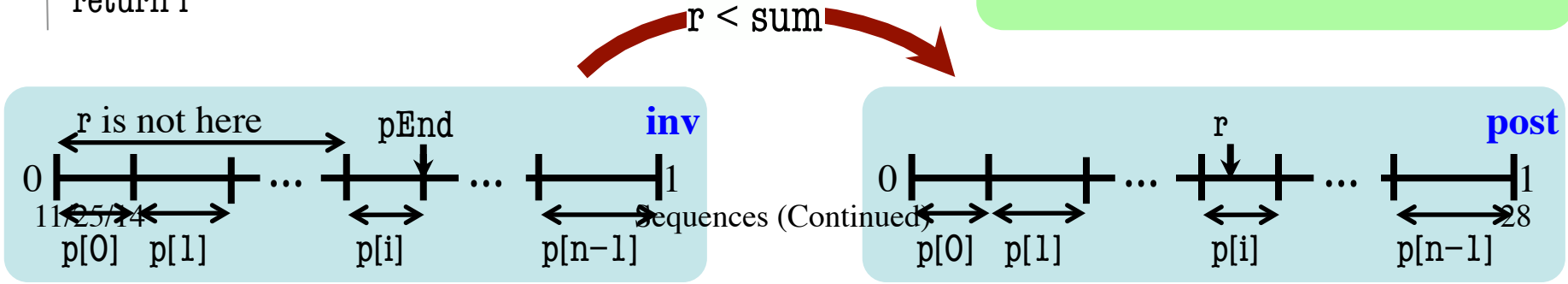

# Reversing a Sequence