Lecture 12

More Recursion

Announcements for This Lecture

Assignments

- A3: Color Models
  - Stage 1 is done
  - Feedback later this week
  - Stage 2 week from Thu.
- Lab 6: Recursion
  - Today's (& Wed) lab
  - Only have to do four
  - Due week after fall break

Prelim 1

- Thursday 7:30-9pm
  - A–Q (Kennedy 1116)
  - R–T (Warren 131)
  - U–Z (Warren 231)
- Graded late Thursday
  - Will have grade Fri morn
  - In time for drop next week
- Make-ups announced

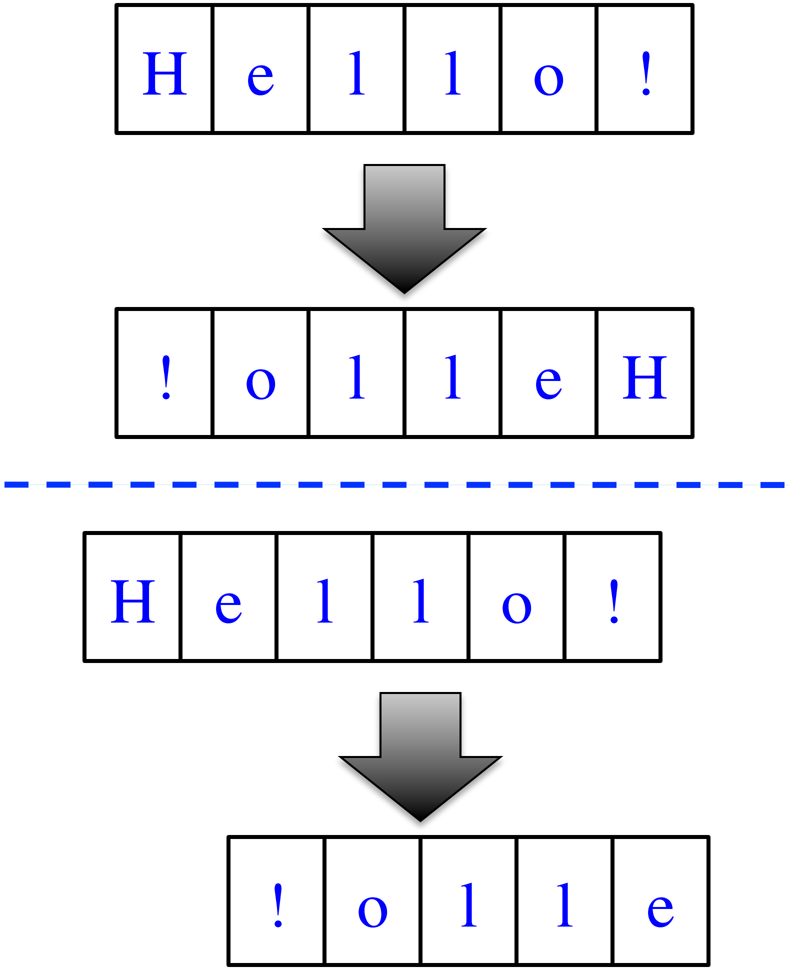
Recursion

- **Recursive Definition**:
  A definition that is defined in terms of itself
- **Recursive Function**:
  A function that calls itself (directly or indirectly)

- Powerful programming tool
  - Want to solve a difficult problem
  - Solve a simpler problem instead
- **Goal of Recursion:**
  Solve original problem with help of simpler solution

Example: Reversing a String

- **Precise Specification**:
  - Returns: reverse of s
- Solving with recursion
  - Suppose we can reverse a smaller string (e.g. less one character)
  - Can we use that solution to reverse whole string?
- Often easy to understand first without Python
  - Then sit down and code



H	e	l	l	o	!

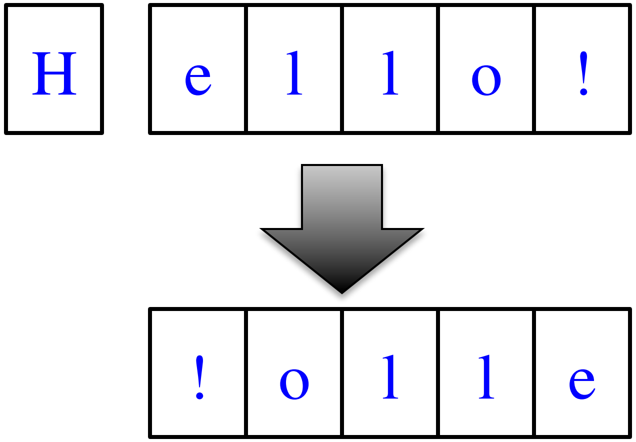
!	o	l	l	e	H

H	e	l	l	o	!

!	o	l	l	e

Example: Reversing a String

```
def reverse(s):
    """Returns: reverse of s

    Precondition: s a string"""
    # {s is empty}
    if s == '':
        return s

    # { s at least one char }
    # (reverse of s[1:])+s[0]
    return reverse(s[1:])+s[0]
```



H e l l o !

! o l l e

✓ 1. Precise specification?

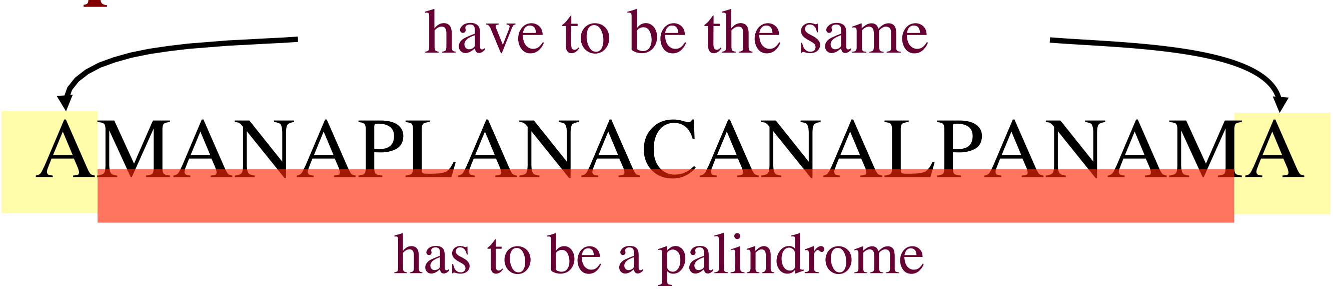
✓ 2. Base case: correct?

✓ 3. Recursive case: progress to termination?

✓ 4. Recursive case: correct?

Example: Palindromes

- String with ≥ 2 characters is a palindrome if:
  - its first and last characters are equal, and
  - the rest of the characters form a palindrome
- **Example:**



have to be the same

AMANAPLANACANALPANAMA

has to be a palindrome

- **Precise Specification:**

```
def ispalindrome(s):
    """Returns: True if s is a palindrome"""
```

Example: Palindromes

- String with ≥ 2 characters is a palindrome if:
  - its first and last characters are equal, and
  - the rest of the characters form a palindrome

Recursive Definition

- **Recursive Function:**

```
def ispalindrome(s):
    """Returns: True if s is a palindrome"""
    if len(s) < 2:
        return True

    // { s has at least two characters }
    return s[0] == s[-1] and ispalindrome(s[1:-1])
```

Base case

Recursive case

Example: Palindromes

- String with ≥ 2 characters is a palindrome if:
  - its first and last characters are e
  - the rest of the characters form

1. Precise specification?
2. Base case: correct?
3. Recursive case: progress to termination?
4. Recursive case: correct?

- **Recursive Function:**

```
def ispalindrome(s):
    """Returns: True if s is a palindrome"""
    if len(s) < 2:
        return True

    // { s has at least two characters }
    return s[0] == s[−1] and ispalindrome(s[1:-1])
```

Base case

Recursive case

Example: More Palindromes

```
def ispalindrome2(s):
    """Returns: True if s is a palindrome
    Case of characters is ignored."""
    if len(s) < 2:
        return True

    // { s has at least two characters }
    return ( equals_ignore_case(s[0],s[−1])
                and ispalindrome2(s[1:-1]) )

def equals_ignore_case (a, b):
    """Returns: True if a and b are same ignoring case"""
    return a.upper() == b.upper()
```

Precise Specification

Example: More Palindromes

```
def ispalindrome3(s):
    """Returns: True if s is a palindrome
    Case of characters and non-letters ignored."""
    return ispalindrome2(depunct(s))


def depunct(s):
    """Returns: s with non-letters removed"""
    if s == '':
        return s
    # use string.letters to isolate letters
    return (s[0]+depunct(s[1:]) if s[0] in string.letters
               else depunct(s[1:]))
```

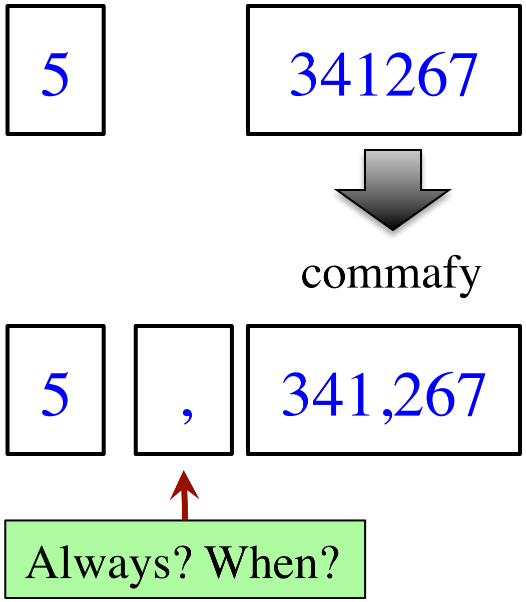
Use helper functions!

- Often easy to break a problem into two
- Can use recursion more than once to solve

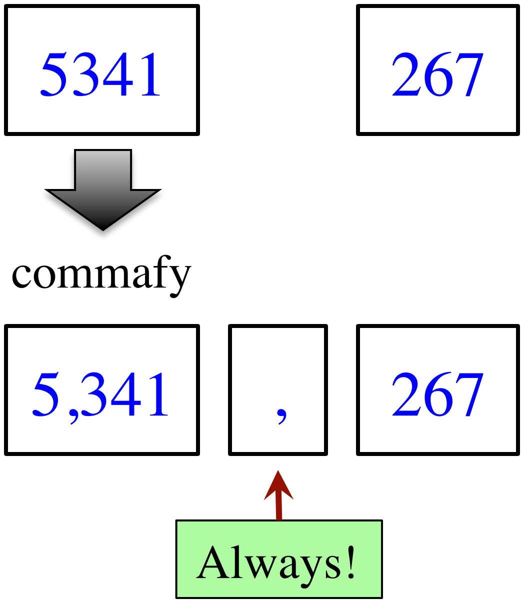
How to Break Up a Recursive Function?

```
def commafy(s):
    """Returns: string with commas every 3 digits
   e.g. commafy('5341267') = '5,341,267'
   Precondition: s represents a non-negative int"""
```

Approach 1



Approach 2



How to Break Up a Recursive Solution?

```
def commafy(s):
    """Returns: string with commas every 3 digits
    e.g. commafy('5341267') = '5,341,267'
    Precondition: s represents a non-negative int"""
    # No commas if too few digits.
    if len(s) <= 3:
        return s

    # Add the comma before last 3 digits
    return commafy(s[:-3]) + ',' + s[-3:]
```

Base case

Recursive case

How to Break Up a Recursive Function?

def exp(b, c)

    """Returns: b^c

    Precondition: b a float, $c \geq 0$ an int"""

Approach 1

$$12^{256} = 12 \times (\boxed{12^{255}})$$

Recursive

$$b^c = b \times (b^{c-1})$$

Approach 2

$$12^{256} = (\boxed{12^{128}}) \times (\boxed{12^{128}})$$

Recursive Recursive

$$b^c = (b \times b)^{c/2} \text{ if c even}$$

Raising a Number to an Exponent

Approach 1

```
def exp(b, c)
    """Returns: b^c
    Precondition: b a float,
                  c ≥ 0 an int"""
    # b^0 is 1
    if c == 0:
        return 1

    # b^c = b(b^c)
    return b*exp(b,c-1)
```

Approach 2

```
def exp(b, c)
    """Returns: b^c
    Precondition: b a float,
                  c ≥ 0 an int""”
    if c == 0:
        return 1
    # c > 0
    if c % 2 == 0:
        return exp(b*b,c/2)

    return b*exp(b*b,c/2)
```

Raising a Number to an Exponent

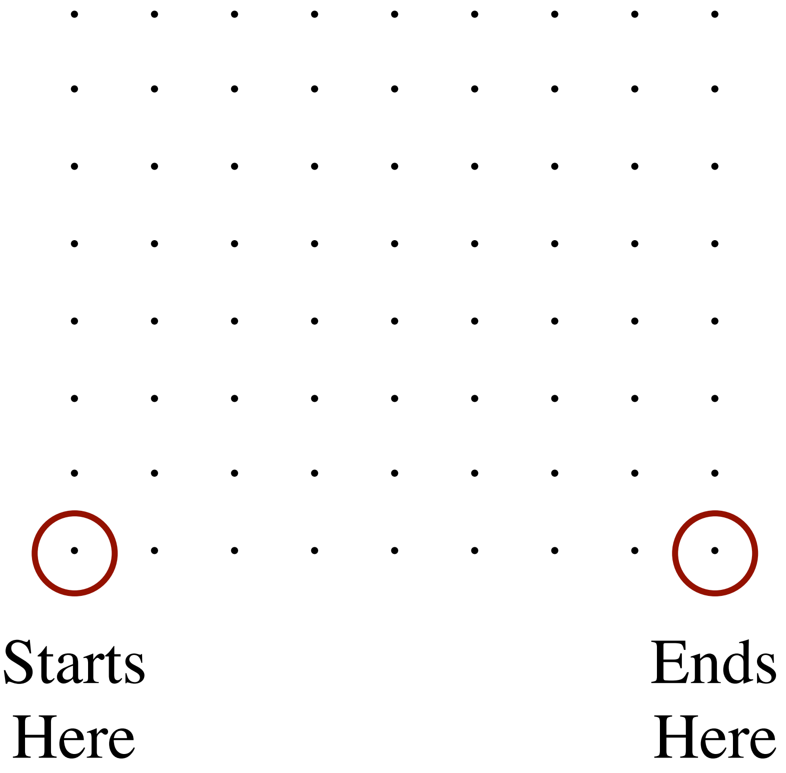
```
def exp(b, c)
    """Returns: b^c
    Precondition: b a float,
                       c ≥ 0 an int"""
    # b^0 is 1
    if c == 0:
        return 1

    # c > 0
    if c % 2 == 0:
         return exp(b*b,c/2)

    return b*exp(b*b,c/2)
```

c	# of calls
0	0
1	1
2	2
4	3
8	4
16	5
32	6
2^n	n + 1

32768 is 215

b^{32768} needs only 215 calls!

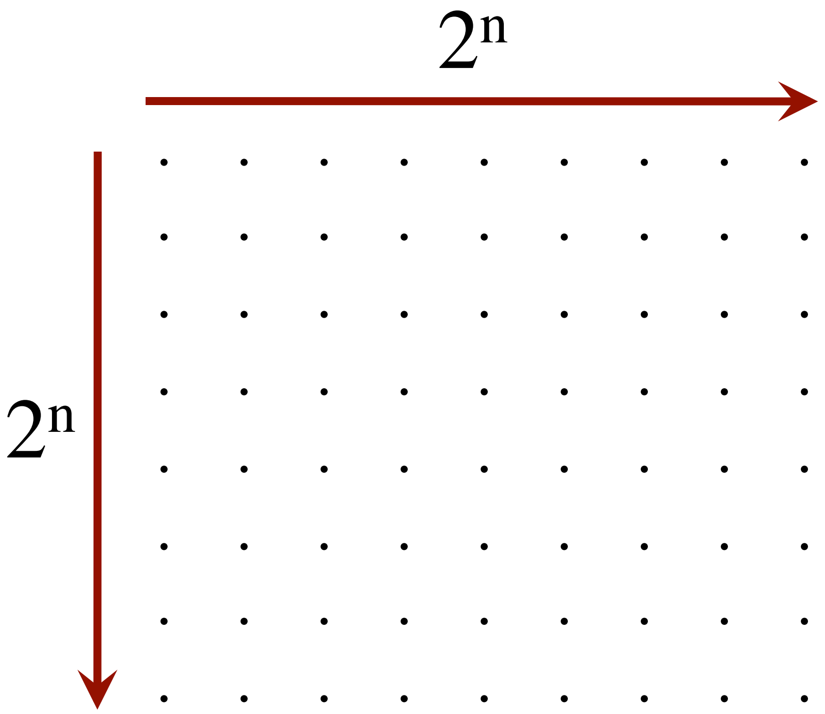
Space Filling Curves



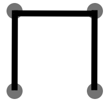
Challenge

- Draw a curve that
  - Starts in the left corner
  - Ends in the right corner
  - Touches every grid point
  - Does not touch or cross itself anywhere
- Useful for analysis of 2-dimensional data

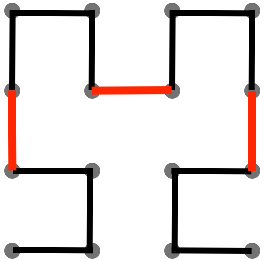
Hilbert's Space Filling Curve



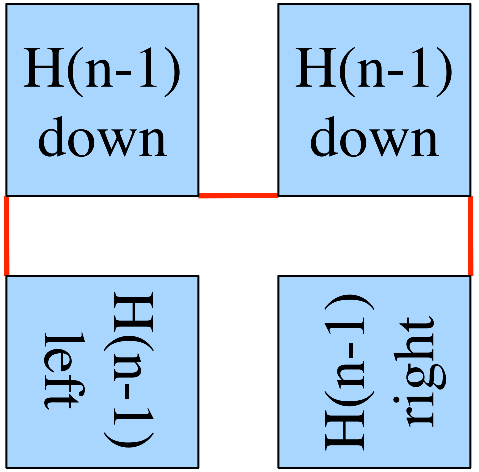
Hilbert(1):



Hilbert(2):



Hilbert(n):



H(n-1) down

H(n-1) down

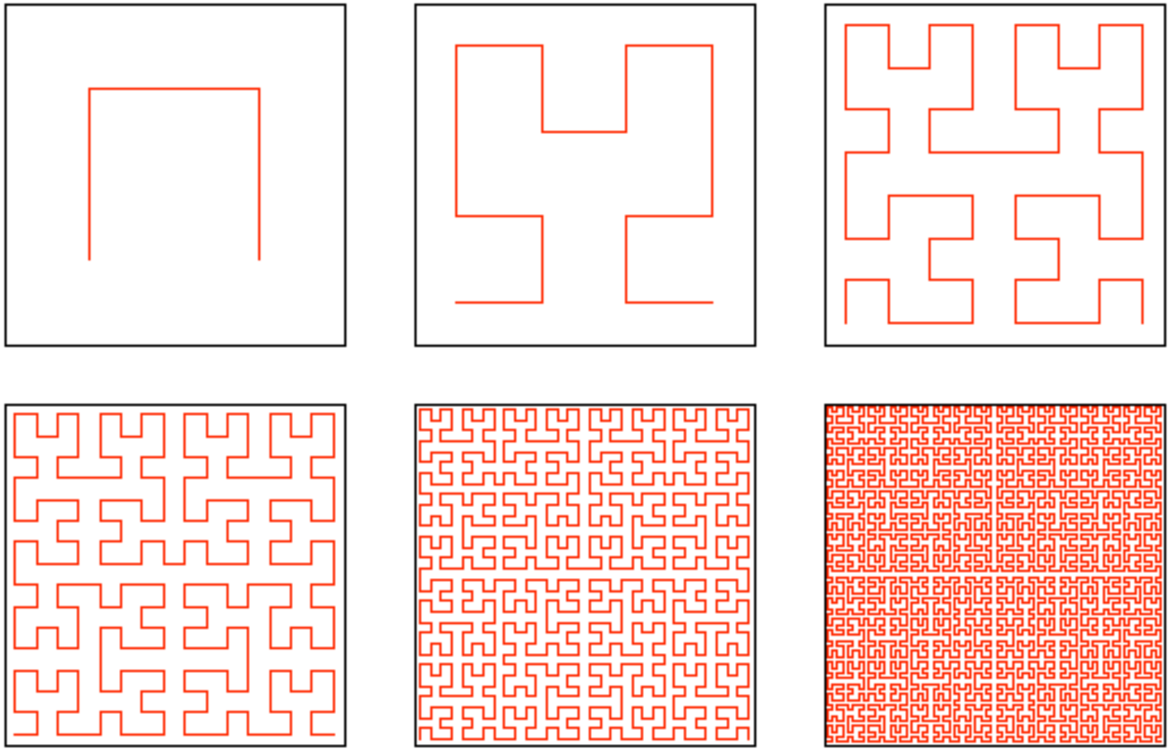
H(n-1) left

H(n-1) right

Hilbert's Space Filling Curve

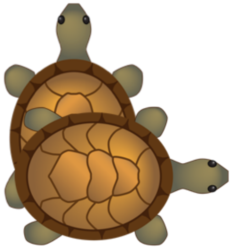
Basic Idea

- Given a box
- Draw $2^n \times 2^n$ grid in box
- Trace the curve
- As n goes to ∞, curve fills box

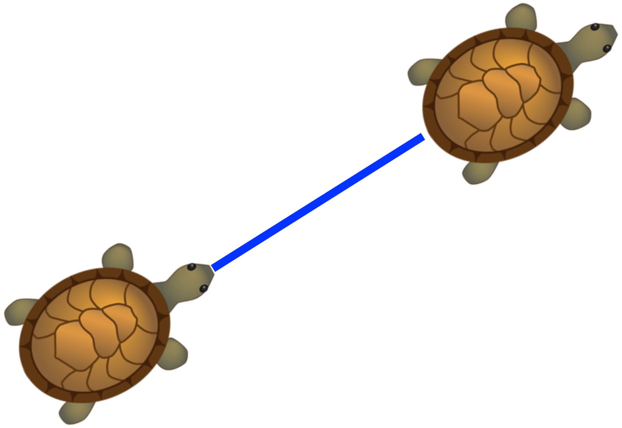


“Turtle” Graphics: Assignment A5

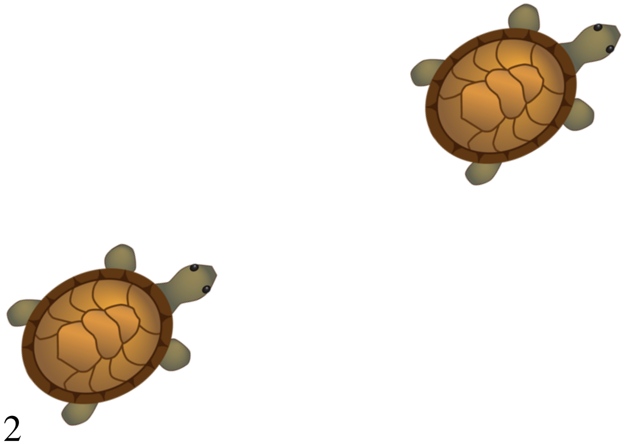
Turn



Draw Line



Move



Change Color