Tricks for Checking Divisibility

2	If the last digit is even, the number is divisible by 2.
3	If the sum of the digits is divisible by 3, the number is also.
4	If the last two digits form a number divisible by 4, the number is also.
5	If the last digit is a 5 or a 0, the number is divisible by 5.
6	If the number is divisible by both 3 and 2, it is also divisible by 6.
7	Take the last digit, double it, and subtract it from the rest of the number; if the answer is divisible by 7 (including 0), then the number is also. For big numbers, alternately add and subtract digits in groups of three. If the answer is divisible by 7, the number is too. For example 256242 is divisible by 7 because 256-242 = 14.
8	If the last three digits form a number divisible by 8, then so is the whole number. Double the hundreds' digit, add the tens' digits, double again and add the ones' digit. If this is divisible by eight, so is the whole number.
9	If the sum of the digits is divisible by 9, the number is also.
10	If the number ends in 0, it is divisible by 10.
11	Alternately add and subtract the digits from left to right. (You can think of the first digit as being 'added' to zero.) If the result (including 0) is divisible by 11, the number is also. *Example:* to see whether 365167484 is divisible by 11, start by subtracting: 3-6+5-1+6-7+4-8+4 = 0; therefore 365167484 is divisible by 11. For big numbers, you can also alternately add and subtract digits in groups of three. If the answer is divisible by 11, the number is too.
12	If the number is divisible by both 3 and 4, it is also divisible by 12.
13	Remove the last digit from the number, then add 4 times the removed digit from the remaining number. If what is left is divisible by 13, then so is the original number. For big numbers, alternately add and subtract digits in groups of three, just like with 7. If the answer is divisible by 13, the number is too.
17	Remove the last digit from the number, then subtract 5 times the removed digit from the remaining number. If what is left is divisible by 17, then so is the original number.
19	Add twice the last digit to the remaining number. If the result is divisible by 19, so is the original number.

See the other side for how to find your own divisibility rules.

How the divisibility tricks work

Several of the rules involve adding the last digit of a number, multiplied by something, to the rest of the number. If we understand why this works, we can come up with our own divisibility rules. Let's think about the rule for 13.

Every number with multiple digits can be split into a "head" H and a tail "T", where the tail is just the ones' digit. For example, with the number 208, the head is H=20 and the tail is T=8. The rule for divisibility by 13 is to compute H+4T, which is 52 in this case. We can apply the rule again to get 5+2x4 = 13, so 208 is divisible by 13.

Why does this work? The original number is really the same thing as 10H+T. This number will be divisible by 13 if the same number multiplied by 4 is divisible by 13: that is, 40H+4T. Now suppose we subtract 39H from this number. We get H+4T. Since 39H is a multiple of 13, H+4T is a multiple of 13 exactly when 40H+4T is a multiple of 13, which is exactly when 10H+T, our original number, is a multiple of 13.

The rule for 7 works similarly. 10H+T is a multiple of 7 if 20H+2T is. Subtracting 21H (a multiple of 7), we get 2T-H. So if 2T-H (or its negative, H-2T) is a multiple of 7, so is 10H+T.

Several of the other divisibility rules work the same way. The rule for 19 works because 19 is one less than 2*10. The rule for 17 works because 3*17 is one greater than 5*10. The rule for 11 works because 11 is one more than 1*10.

Can you use this idea to make your own divisibility rule for 29? 31? 23?

For 7, 11, and 13, there is the additional trick of grouping numbers into groups of 3 and alternately adding and subtracting them. This works because alternately adding and subtracting numbers in groups of three gives the remainder when dividing the number by 1001. Since $1001 = 7\times11\times13$, that remainder is divisible by 7, 11, or 13 exactly when the original number is.

Other numbers next to powers of 10 can be used similarly. For example, *adding* the digits in groups of three gives the remainder when dividing by $999 = 3^3\times37$, and $10001 = 73\times137$. But 1001 is the most useful one to know other than 9 and 11.