Bounding Consideration Probabilities in Consider-Then-Choose Ranking Models

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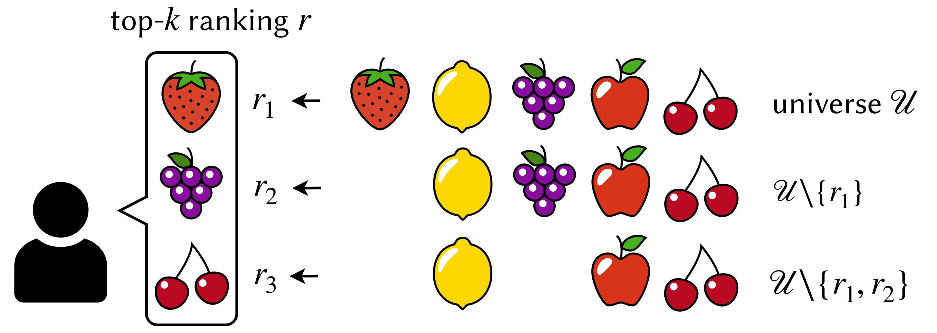
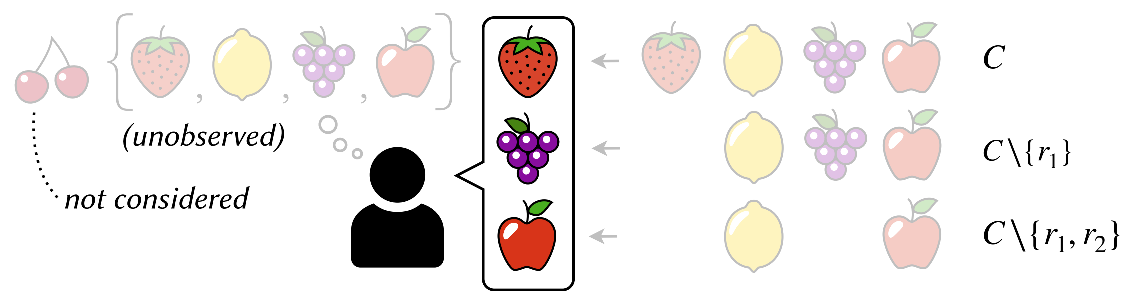
Background

Choice-based ranking



Consider-then-choose ranking



Consider-then-choose model: select $C \subseteq \mathcal{U}$ to consider, then rank k elements from C (common in discrete choice, rarely applied to rankings)

Overview

consideration set model

$$\Pr_C(C) = 0 \quad \text{if } |C| < k$$

$$\Pr_C(C) \propto \left(\prod_{i \in C} p_i\right) \prod_{j \in \mathcal{U}\setminus C} (1 - p_j)$$

PL with Consideration (PL+C)

1. People consider at least k items
2. Each item i is considered independently w.p. p_i
3. Given C, items are ranked by Plackett-Luce

Plackett-Luce model [1, 2]

$$\Pr_{PL}(r \mid C) = \prod_{i=1}^{k} \frac{\exp(u_{r_i})}{\sum_{j \in \mathscr{C}\setminus\{r_{1\ldots i-1}\}} \exp(u_j)}$$

$$\Pr_{PL+C}(r) = \sum_{C \subseteq \mathcal{U}} \Pr_C(C) \Pr_{PL}(r \mid C)$$

Can we tell from rankings what items people consider?

We show this is impossible in general. But we provide:

1. **Relative bounds** on consideration probabilities, given known item utilities.
2. **Absolute bounds** on consideration probabilities, given utilities and a lower bound on expected number of items considered.
3. An **efficient algorithm** to tighten our absolute bounds using our relative bounds.

Theory

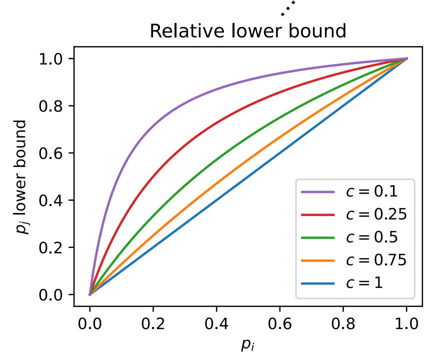
General impossibility of learning consideration

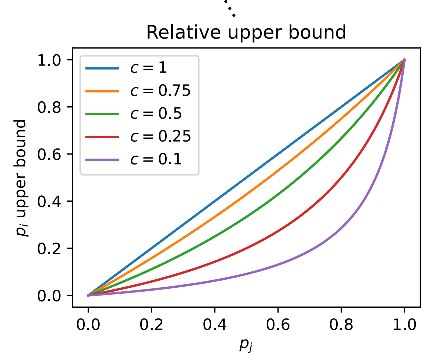
Theorem 1. PL+C consideration probabilities are not identifiable, even if we know item utilities.

Relative bounds on consideration probabilities

Theorem 2. If $u_i > u_j$, but i is ranked in the top-ℓ $c \leq 1$ times as often, then $\dfrac{p_i}{1 - p_i} \leq c \cdot \dfrac{p_j}{1 - p_j}$.

Equivalently, $p_i \leq \dfrac{cp_j}{1 - p_j + cp_j}$ and $p_j \geq \dfrac{p_i}{c - cp_i + p_i}$.





Absolute bounds on consideration probabilities

Theorems 3+4. If $\sum_{i \in \mathcal{U}} p_i \geq \alpha k$ for $\alpha > 1$, then

$$\Pr_{PL+C}(\mathscr{R}_{i \leq k}) \cdot \left[1 - \left(\alpha e^{1-\alpha}\right)^k\right] \leq p_i \leq \frac{\sum_{j \in \mathcal{U}} \exp(u_j)}{\exp(u_i)} \cdot \left(\Pr_{PL+C}(\mathscr{R}_{i=1}) + \frac{k(\alpha e^{1-\alpha})^k}{1 - (\alpha e^{1-\alpha})^k}\right).$$

Pr[i appears in top-k rankings] · *correction (Chernoff)* · *1 / Pr[i ranked first] (ignoring consideration)* · *Pr[i ranked first]* · *correction (Chernoff)*

absolute lower bound
i must have been considered whenever it's ranked
+ correction for $|C| \geq k$ conditioning

absolute upper bound
if i was ranked first less often than we expected, it must not have been considered
+ correction for $|C| \geq k$ conditioning

Efficient propagation algorithm

Given: utilities u_i, top-ℓ ranking probabilities $\Pr_{PL+C}(\mathscr{R}_{i \leq \ell})$, and $\alpha > 1$ s.t. $\sum_{i \in \mathcal{U}} p_i \geq \alpha k$ ($\geq \alpha k$ items considered on average).

1. Initialize upper/lower bounds according to Theorem 3/4
2. Construct DAG G of all item reversals (utility vs top-ℓ ranking probability)
3. Propagate bounds using Thm. 2 along a topological sort of G

Experiments

US history perception dataset [3]

Survey asked ~2900 Americans:

Top-3 Question: What three states contributed most to U.S. history?
ranking with consideration

Random-10 Question: What percentage of U.S. history did [10 random states] contribute?
ranking without consideration

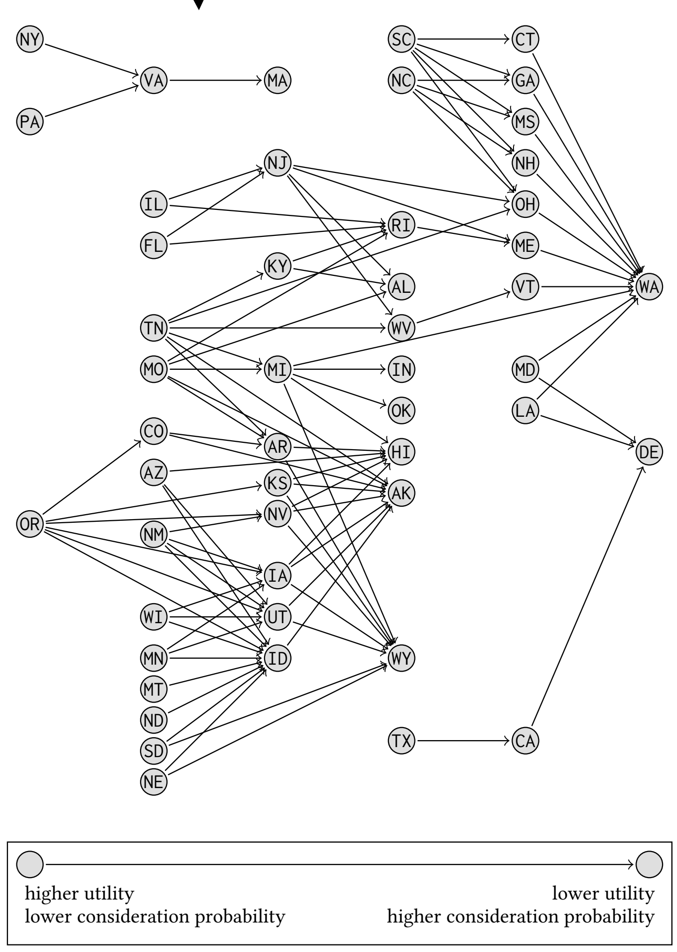
learn utilities from Random-10

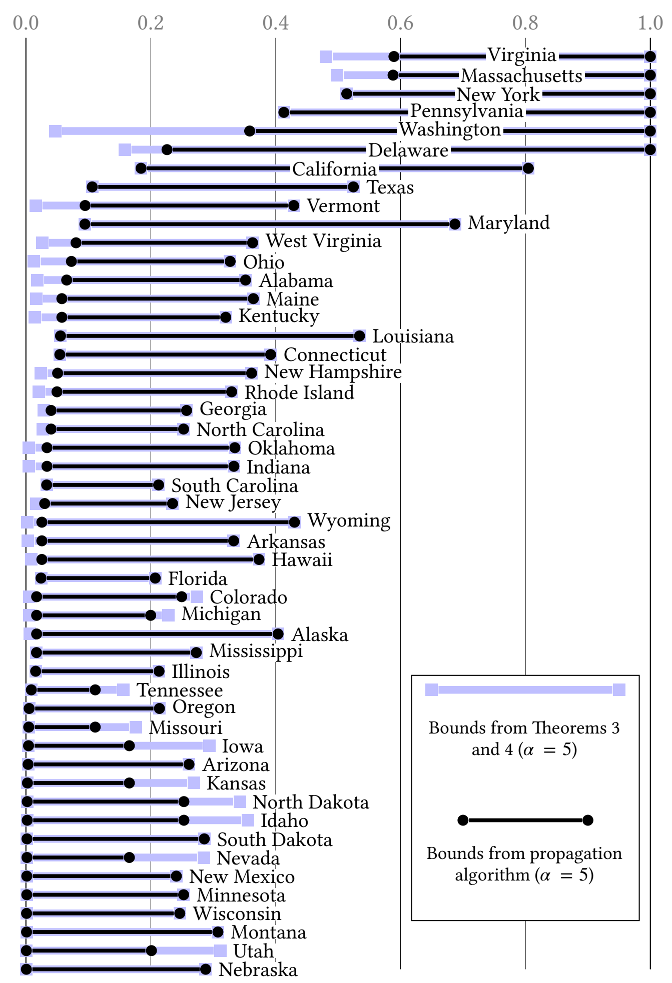
↓

apply propagation algorithm to Top-3 with $\alpha = 5$ (assume 15+ states considered on average)

↓

bounds on how often each state was considered





References

[1] Plackett. The analysis of permutations. *J R Stat Soc Ser C Appl Stat*, 1975.
[2] Luce. *Individual Choice behavior: A theoretical analysis*, Wiley 1959.
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