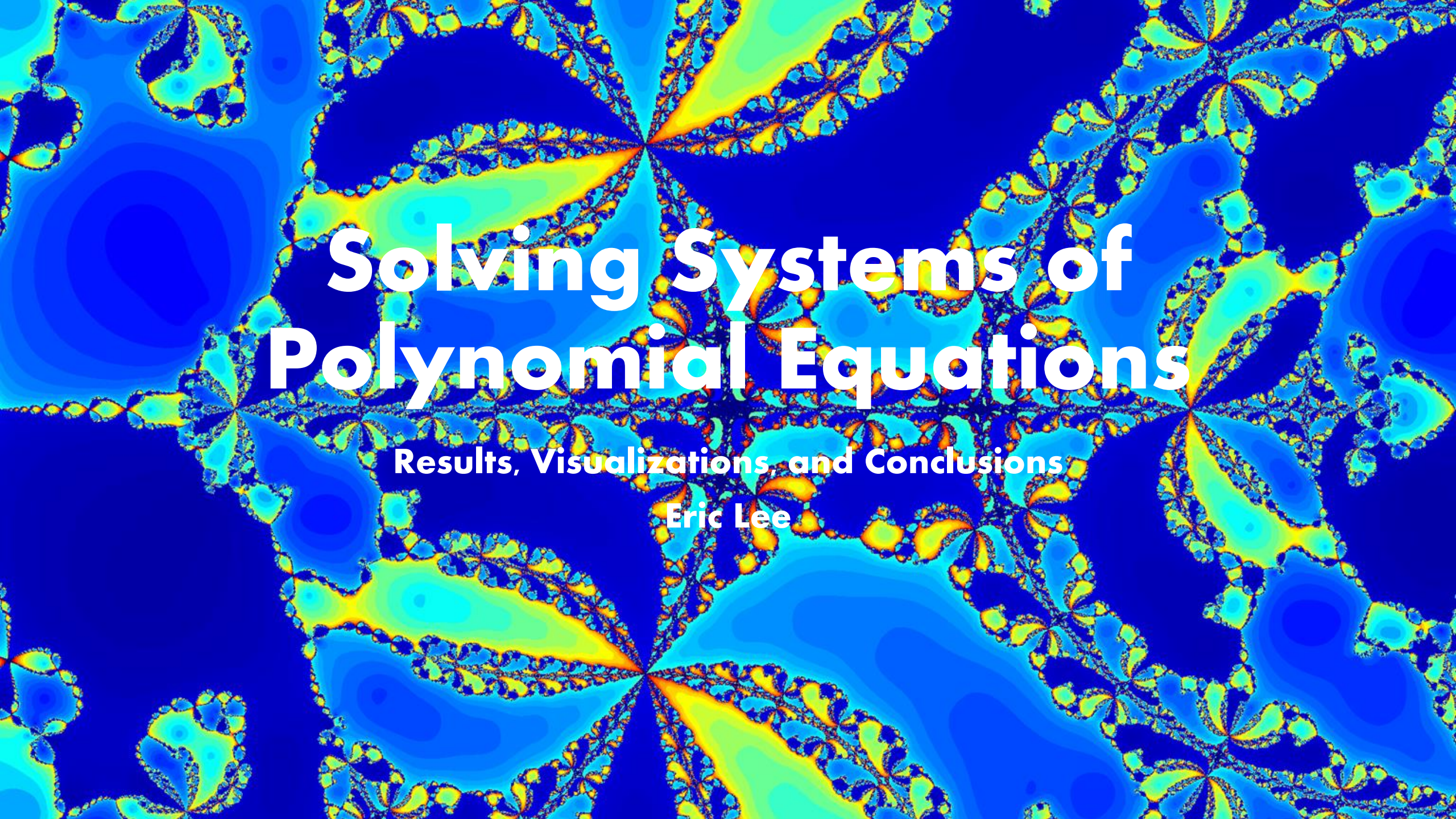


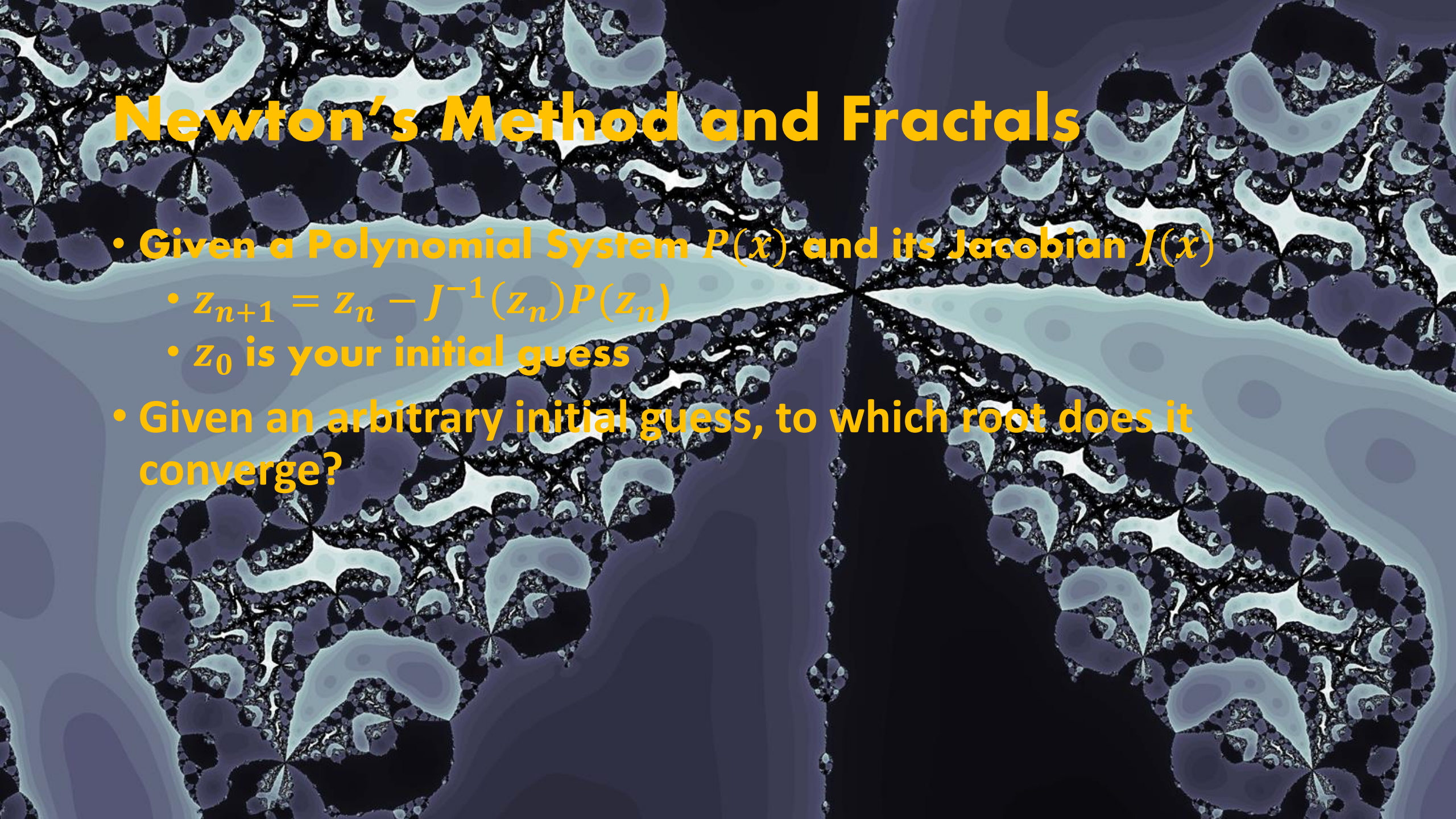
Solving Systems of Polynomial Equations

Results, Visualizations, and Conclusions

Eric Lee

Existing Methods

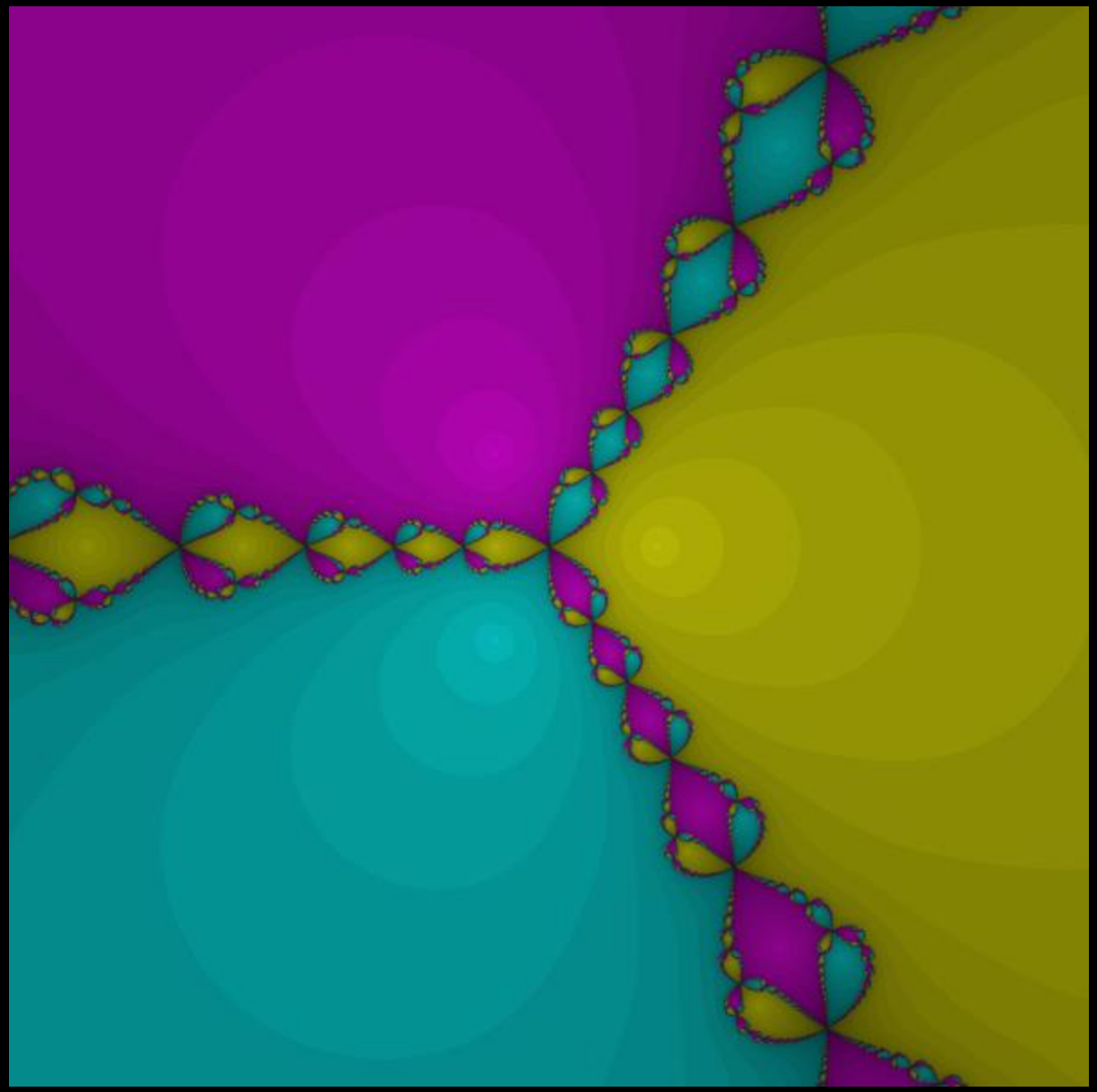
- Groebner Bases (Analytic Solutions)
- Newton's Method (Approximate Solutions)
- Homotopy Methods (Approximate Solutions)
- Optimization (Approximate Solutions)
- All of these are flawed in some way or another!



Newton's Method and Fractals

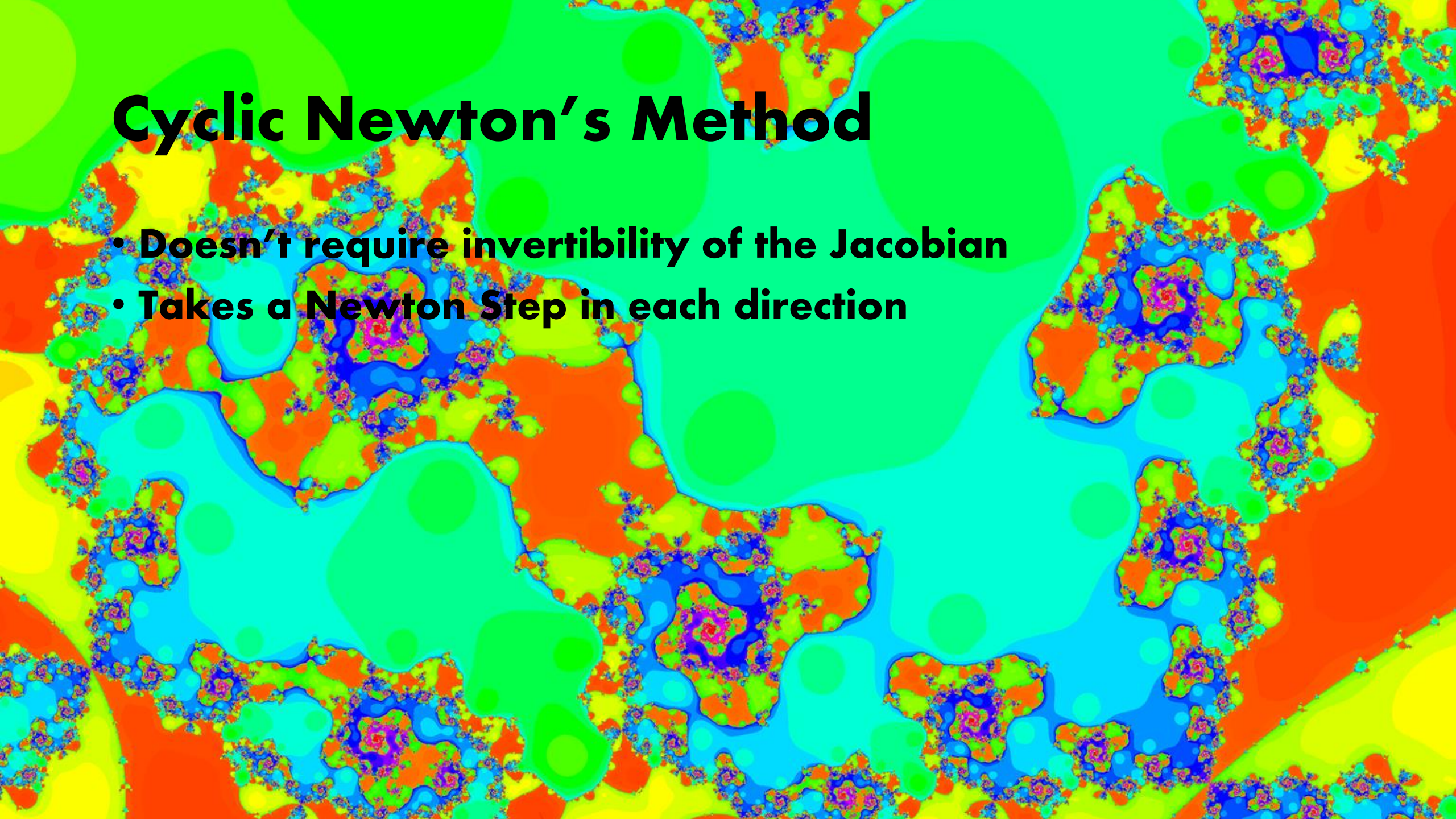
- Given a Polynomial System $P(x)$ and its Jacobian $J(x)$
  - $z_{n+1} = z_n - J^{-1}(z_n)P(z_n)$
  - z_0 is your initial guess
- Given an arbitrary initial guess, to which root does it converge?

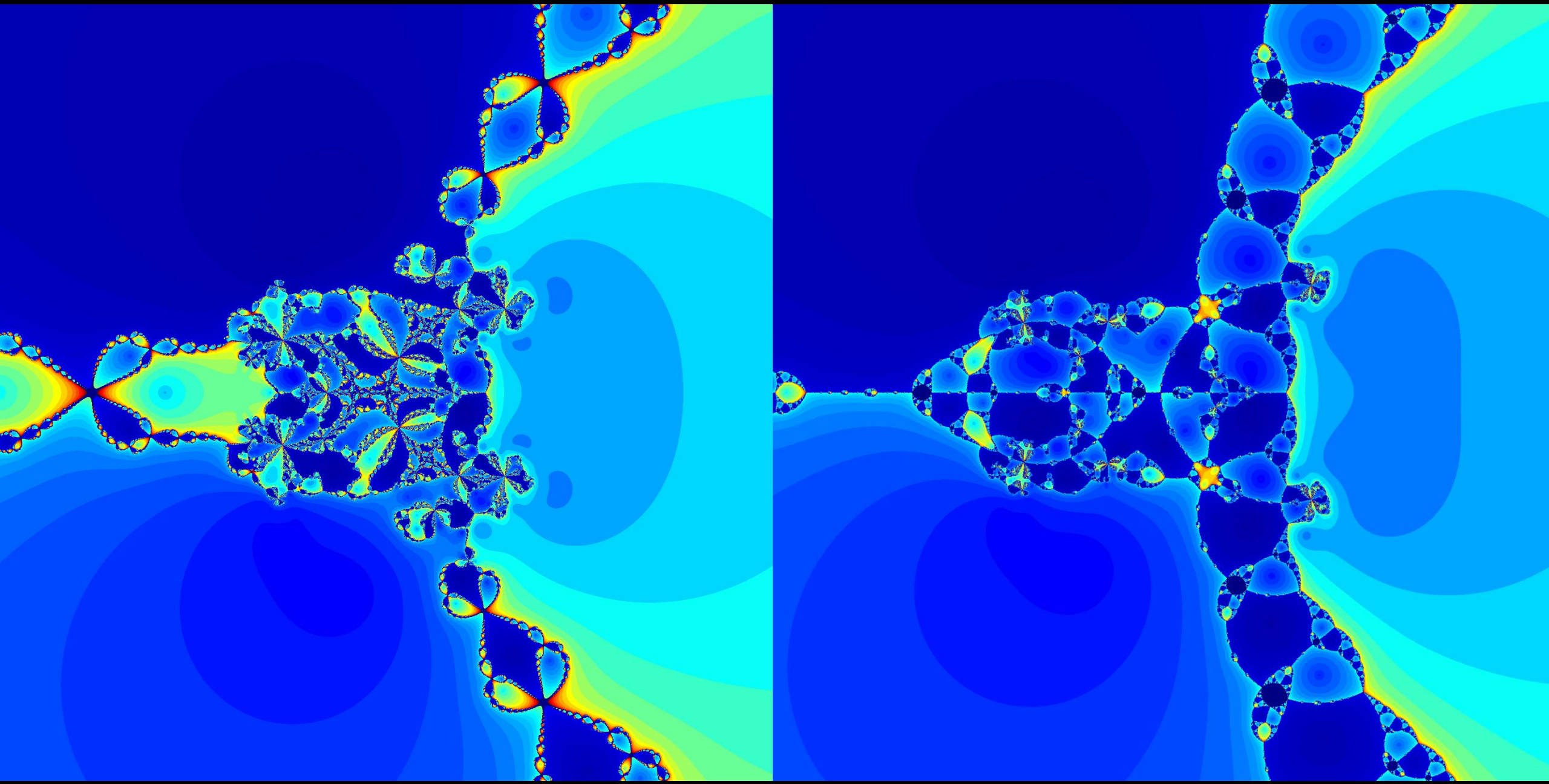
$z^3 - 1$



Cyclic Newton's Method

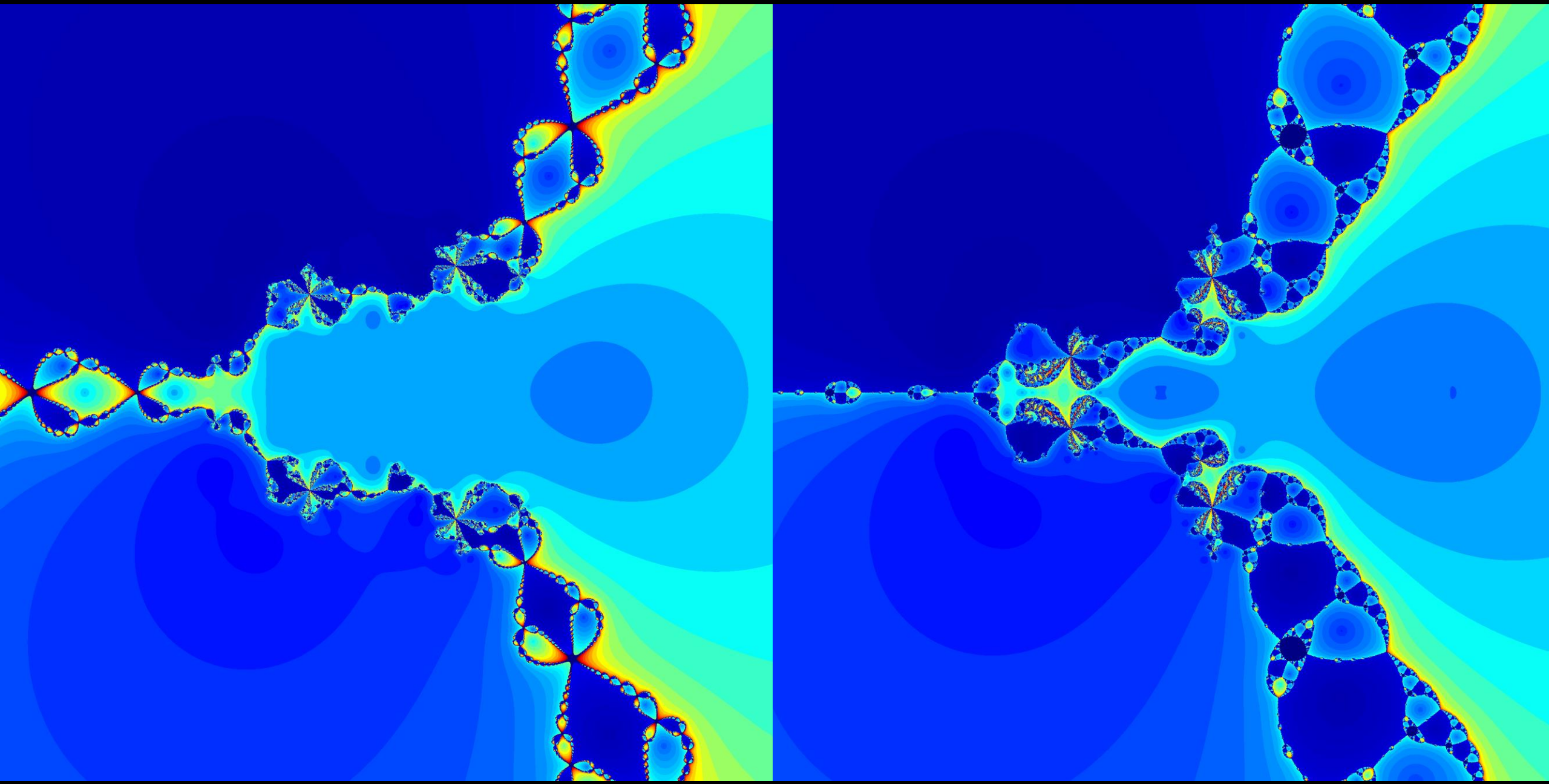
- Doesn't require invertibility of the Jacobian
- Takes a Newton Step in each direction





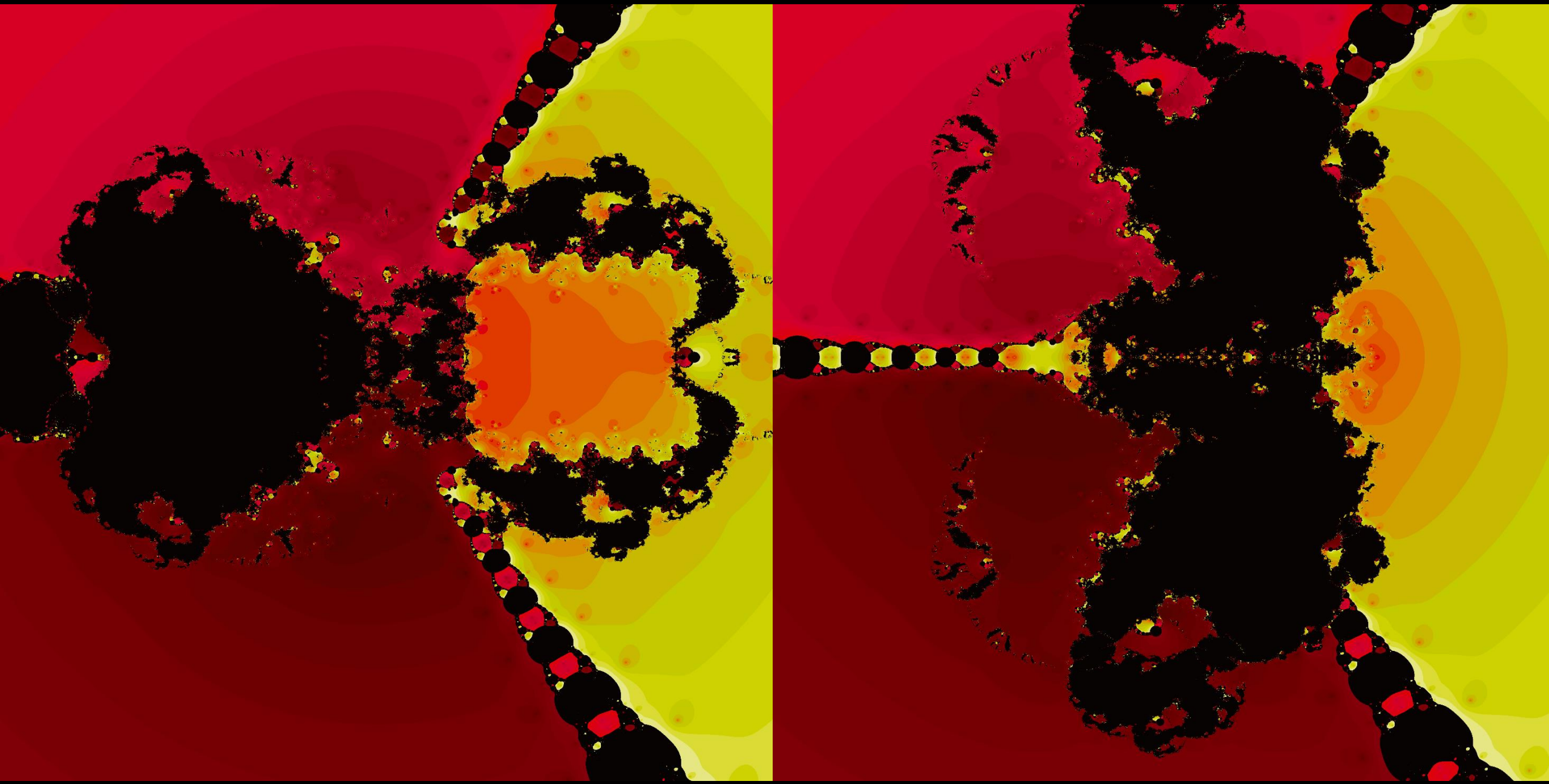
Heuristics

- Dynamical Systems
  - Iterations enter circular orbits around roots or explode to infinity
  - To fix it, we take midpoint steps when the sign of the gradient changes drastically
  - This leads to much more predictable convergence
- Initial Guesses
  - Take initial guesses on the surface of one or two equations



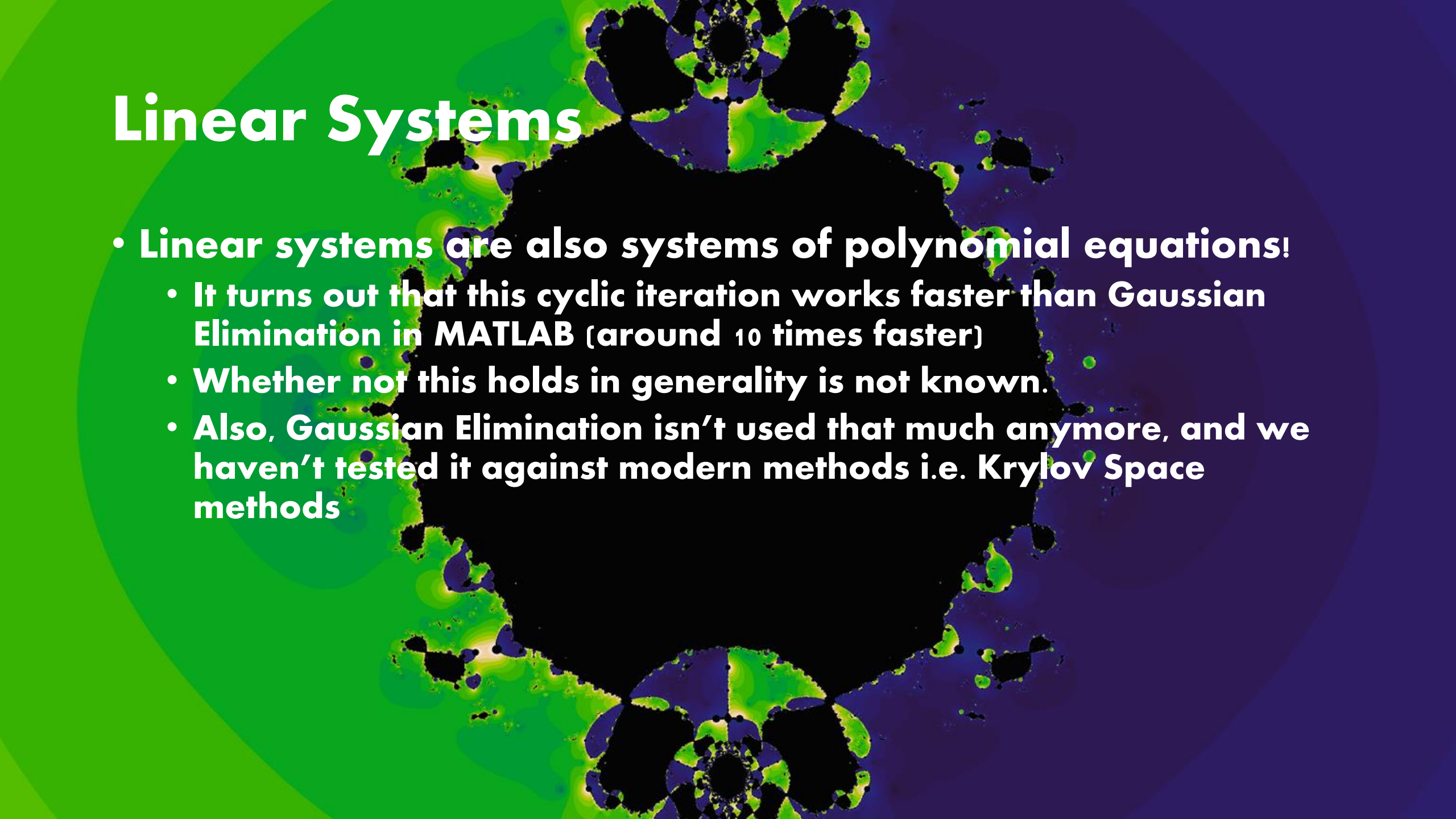
A classes of cyclic iteration methods

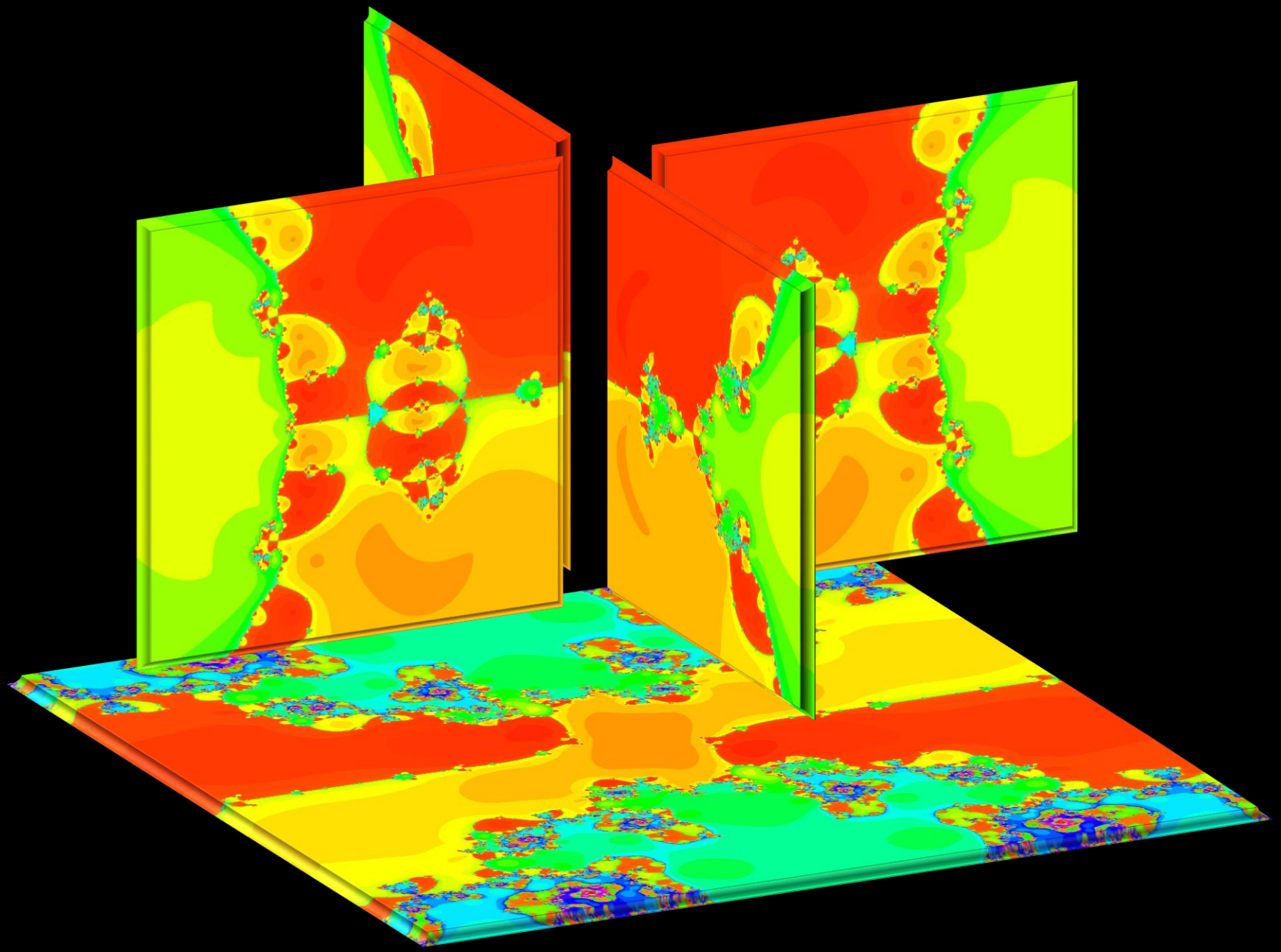
- We can use higher order convergence methods, i.e. Halley's method or Dr. Kalantari's Basic Family (which have order of convergence 3 and 4...n respectively)
- We can also change up the order of alternation



Linear Systems

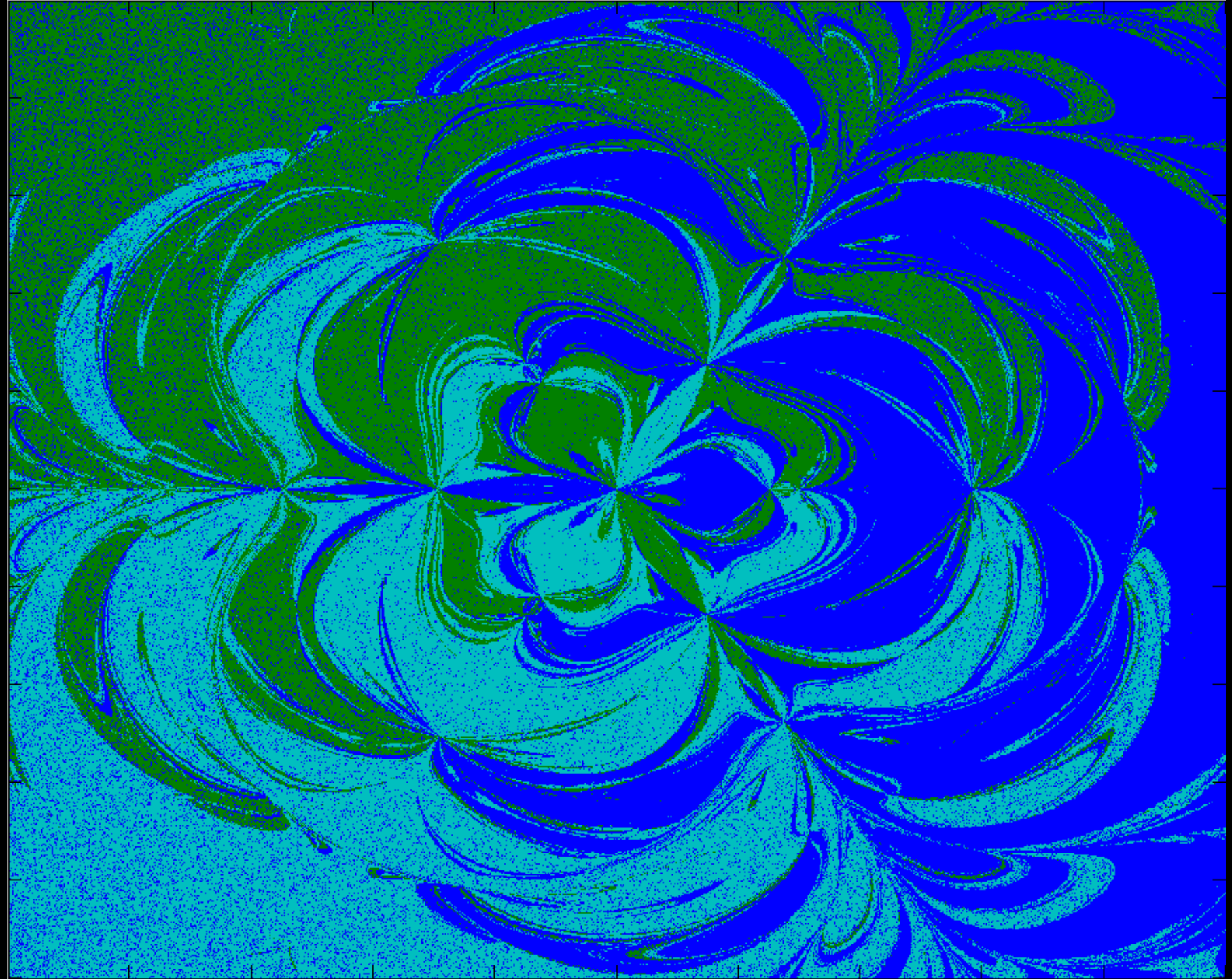
- Linear systems are also systems of polynomial equations!
  - It turns out that this cyclic iteration works faster than Gaussian Elimination in MATLAB (around 10 times faster)
  - Whether not this holds in generality is not known.
  - Also, Gaussian Elimination isn't used that much anymore, and we haven't tested it against modern methods i.e. Krylov Space methods





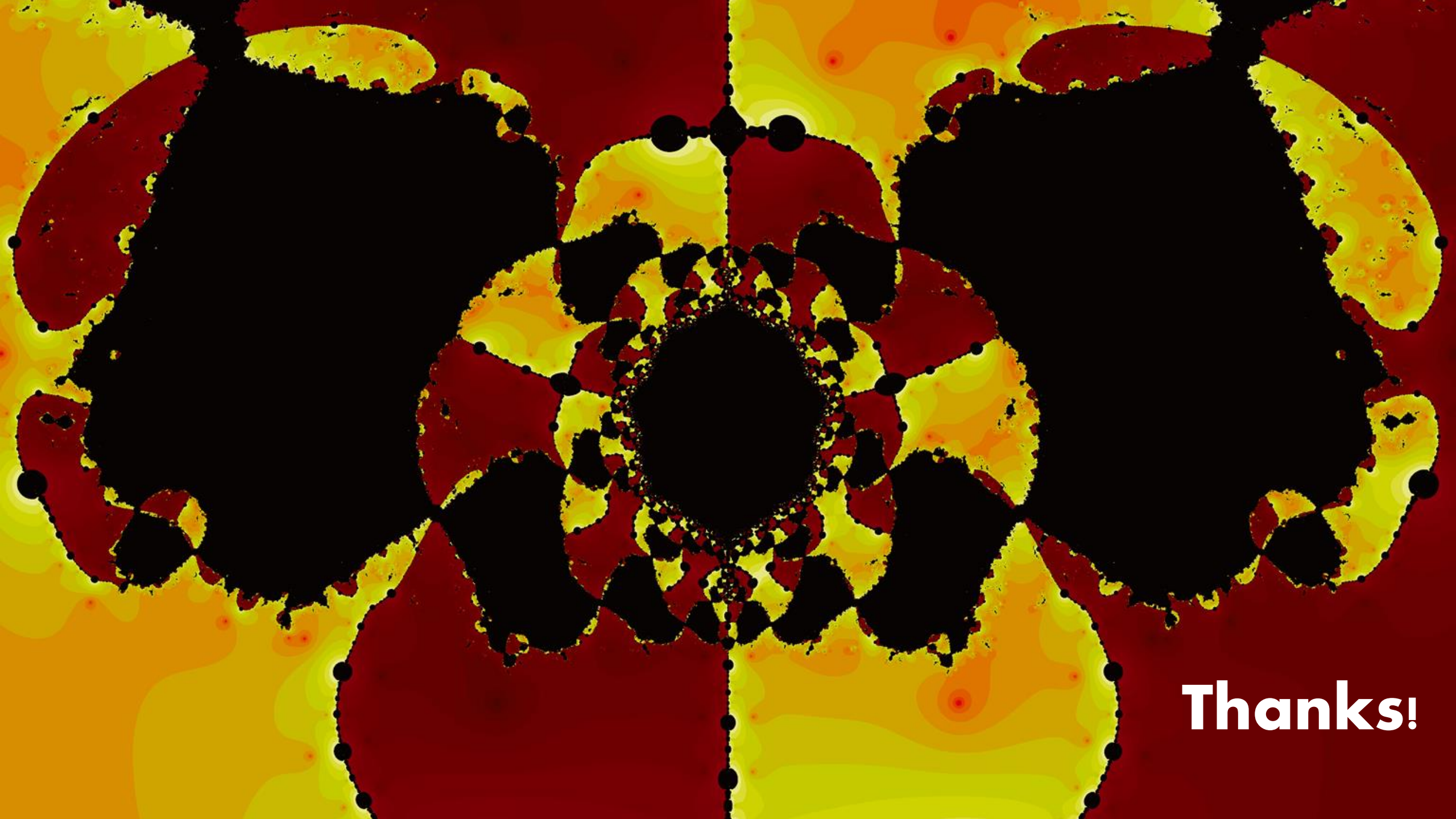
Newton-Ellipsoid Methods

- Sort of like binary search/bisection method
- Given a search space, cut it by a factor until search space is the desired size
- Works for the monovariate polynomials (i.e. one single variable polynomial). We are working on generalizing it



Conclusions

- Cyclic Iteration Methods work well, better and faster in many instances than existence methods
- The Newton-Ellipsoid Method is an excellent way of solving polynomial equations; while the behavior is much more erratic it converges to a root almost everywhere
- Solving nonlinear systems


Thanks!