Theory Qualifier
January 19, 1999

Instructions

Please read these instructions and all questions C A R E F U L L Y before proceeding.

You will have $2\frac{1}{2}$ hours. The exam is closed-book. All questions are of equal weight. You must attempt to answer all of them to earn an unconditional pass. All algorithms should be accompanied by a proof of correctness and complexity analysis unless otherwise directed.

The questions will be graded according to the following criteria in order of importance:

1. correctness and completeness
2. clarity, precision, and conciseness
3. optimality of result

Partial credit will be awarded where appropriate, so be sure to show all work. Unclear or irrelevant arguments will be penalized.

GOOD LUCK!

1. Consider the following clustering problem on bipartite graphs. You are given a bipartite graph $G = (V, E)$ and an integer k. You wish to decide whether there exists a set of edges $F \subset E$ of at size at most k such that there are no isolated vertices in the graph (V, F). Give a polynomial-time algorithm for this problem.

2. Design an algorithm that, given a sequence of n real numbers $x_1, \ldots, x_n$, finds i and j maximizing $\sum_{k=i}^{j} x_k$. Full credit will only be given for a linear-time algorithm.

3. One of the following questions is decidable and the other is undecidable. Say which is which and give proof for both. Why doesn't this contradict Rice's theorem?

   (a) Given a Turing machine M, does there exist a Turing machine N such that $|L(N)| = 2 \cdot |L(M)|$?

   (b) Given a Turing machine M, does there exist a Turing machine N such that $|L(M)| = 2 \cdot |L(N)|$?

4. Show that the following problem is NP-complete. Given a directed graph G with an arbitrary integer weight associated with each edge, does G contain a simple cycle C such that the sum of the edge weights on C is equal to 0?

End of Exam