Applied Logic - CS4860-2018 - Problems Set 2

Abstract

This is a simple problem set which Siva and I would like to grade during the short February break; so we ask that it be handed in during Lecture 7 on Thursday Feb 15, the day after Valentine's Day.

1. On page 24 Smullyan lists eight classical tautologies. We covered the first three in lecture. For the last five, items (4) to (8), please give constructive proofs of any proposition that is constructively valid. Use the refinement rules from Lectures 5 and 6.
2. Explain informally the *computational content* of the evidence for the propositions that are probable in the previous problem.
3. For any of the propositions among (4) to (8) which is not constructively valid, give a classical tableaux proof and explain why it is not constructive in the style used in Lecture 6.
4. Consider Pierce's Law, $((P \Rightarrow Q) \Rightarrow P) \Rightarrow P$. Explain why this proposition is **not** constructively true.
5. If we add the hypothesis $(Q \vee \neg Q)$ to Pierce's Law, give a constructive proof, and explain the evidence semantics.

1 Examples

1. $\vdash \neg\neg\neg A \Rightarrow \neg A$ by $\lambda(f.\ \_\_\_\_\_\ )$

   $f : (((A \Rightarrow \bot) \Rightarrow \bot) \Rightarrow \bot) \vdash\ A \Rightarrow \bot$ by $\lambda(a.\_\_\_\_\_)$

   a: A $\vdash \bot$ by $ap(f;\_\_\_)$ nothing left to do but apply f and this will achieve the goal, $\bot$.

   $\vdash (A \Rightarrow \bot) \Rightarrow \bot$ by $\lambda(na.\_\_\_\_\_)$

   $na : A \Rightarrow \bot\ \vdash \bot$ by $ap(na; a)$

   Extract: $\lambda(f.\lambda(a.ap(f;\lambda(na.ap(na;a)))))$

2. $\vdash \neg A \Rightarrow \neg\neg\neg A$ by $\lambda(na.\ \_\_\_\_\_\ )$

   $na : \neg A \vdash (((A \Rightarrow \bot) \Rightarrow \bot) \Rightarrow \bot)$ by $\lambda(nna.\_\_\_\_\_)$

   $nna : ((A \Rightarrow \bot) \Rightarrow \bot) \vdash \bot$ by $ap(nna; na)$

   $\vdash (A \Rightarrow \bot) \Rightarrow \bot$ by $\lambda(na.\_\_\_\_\_)$

   $na : A \Rightarrow \bot\ \vdash \bot$ by $ap(na; a)$

   Extract: $\lambda(na.\lambda(nna.ap(nna;na))$

3. $\vdash \neg\neg(P \vee \neg P)$
   This expands to

$((P \vee \neg P) \Rightarrow \bot) \Rightarrow \bot$ by $\lambda(f.\ \_\_\_\_\_\_)$

$f : ((P \vee \neg P) \Rightarrow \bot) \vdash \bot$ by $ap(f; \_\_\_))$

$\vdash (P \vee \neg P)$ by $inr(\_\_\_\_\_\_)$

$\vdash \neg P$ by $\lambda(p.\_\_\_\_\_\_)$

$p : P \ \vdash \bot$ by $ap(f; \_\_\_)$

$\vdash (P \vee \neg P)$ by $inl(p)$

$\vdash P$ by p

$v2 : \bot \ \vdash \bot$ by v_2

$v1 : \bot \ \vdash \bot$ by v

Extract: $\lambda(f.ap(f; inr(\lambda(p.ap(f; inl(p))))))$